



## PRIMES: How to Succeed in Mathematical Research



*Professor Pavel Etingof, the Chief Research Advisor of PRIMES, gives his advice.*

A recipe for succeeding in mathematical research would have been a laughable oxymoron, as this is a quintessentially creative endeavor. Yet, I'd like to offer a few tips for those who are taking their first steps along this fascinating path.

1. Be stubborn and at the same time flexible. In mathematical research, unlike olympiads, solving a problem takes weeks and months rather than hours, and there is no instant gratification. Yet, you don't want to rack your brains for too long without progress if you get stuck. Ask for help, or switch to another problem!
2. Be knowledge-seeking. If you did not make progress on your problem but learned something instead, then you did in fact make progress. Besides being intellectually rewarding by itself, at the end of the day learning always pays off in practical terms as well -- it helps you obtain better results. In fact, look for directions of study that will force you to learn something! The more you learn to enjoy the process of doing mathematics (rather than the result), the better mathematician you will be.
3. Mathematical research is an intrinsically social activity. Discuss a lot, seek help/advice/feedback from others, rather than be stuck for a long time.
4. Split the problem into small, bite-size steps, or ask your mentor to do so for you. You want to have something doable on your agenda at all times.
5. Consider examples. Look for the simplest example that captures the phenomenon ([Gelfand's principle](#)). Also tracing through the proof with the simplest nontrivial example is a great way of checking a proof. It often uncovers subtle errors that are harder to see in the more general context, or ways to drastically simplify the proof, by identifying parts which are a "red herring."
6. Have several questions to think about so that you can switch from one to the other.
7. Use the Internet ([Wikipedia](#) is usually good for math, even though you have to be careful with it). Also [Google](#) (or [Google Scholar](#)) keyword search is often helpful. But you have to know the right keywords, and it sometimes takes some thinking to come up with them! Also, a good source is [MathOverflow](#), where you may ask a question and professional mathematicians will answer it online. But make sure that your question is well stated, according to the rules of MathOverflow!
8. Use analogies. There are many mathematical problems but much fewer methods for solving them. So a method used in one problem may also work in another, analogous one.
9. Do computer tests, look for patterns in data, make conjectures. [The On-Line Encyclopedia of Integer Sequences](#) is often useful. It has an advanced tool called [Superseeker](#) which seeks patterns.
10. Confirm your results and proofs by computer calculations whenever possible, to avoid mistakes.
11. Keep good notes of what you are doing (preferably in [LaTeX](#)) at all times. Good bookkeeping is a big part of doing math!
12. Try to write clearly and concisely, in logical sequence. A mathematical text should be highly structured. A good way of writing, at least for beginners, is to make sure that each piece of text is a definition, proposition, proof, remark, example, question, conjecture, etc., so that the text is split into small pieces and there is little (if any) loose text that does not fall into one of these categories. About each piece of text it should be clear what its status is. Text should be proofread and edited several times after it's written.
13. Try to understand statements and proofs of the results that you use as well as you can. Not only is it more honest and reliable, but this will also give you more power in handling the actual problem you deal with.
14. Be motivated and guided by beauty and harmony. It is the most important motivation in mathematics. If you have a proof but don't like it, if it seems ugly, it is much more likely that it is actually wrong. And even if it's correct, it probably will become much simpler or more powerful, and you'll learn something if you try to understand things better so you can write a better proof. It is worth trying to understand better the things you already understand to some extent, rather than jumping forward to entirely new things. Although this seemingly slows down the process, you will surely make up for it and be rewarded further down the road.
15. Listen to your heart. As in all important things in life, what you want and what you dream about is the most essential. Try to find your own voice. The main point of mathematical research is for you to enjoy it!

**Email us:** [primes@math.mit.edu](mailto:primes@math.mit.edu)