INDIAN INSTITUTE OF SCIENCE EDUCATION AND RESEARCH (IISER), PUNE
(An Autonomous Institution, Ministry of Human Resource Development, Govt. of India)
Dr Homi Bhabha Road, Pune − 411 008

August 2025 : Mid Semester Examination

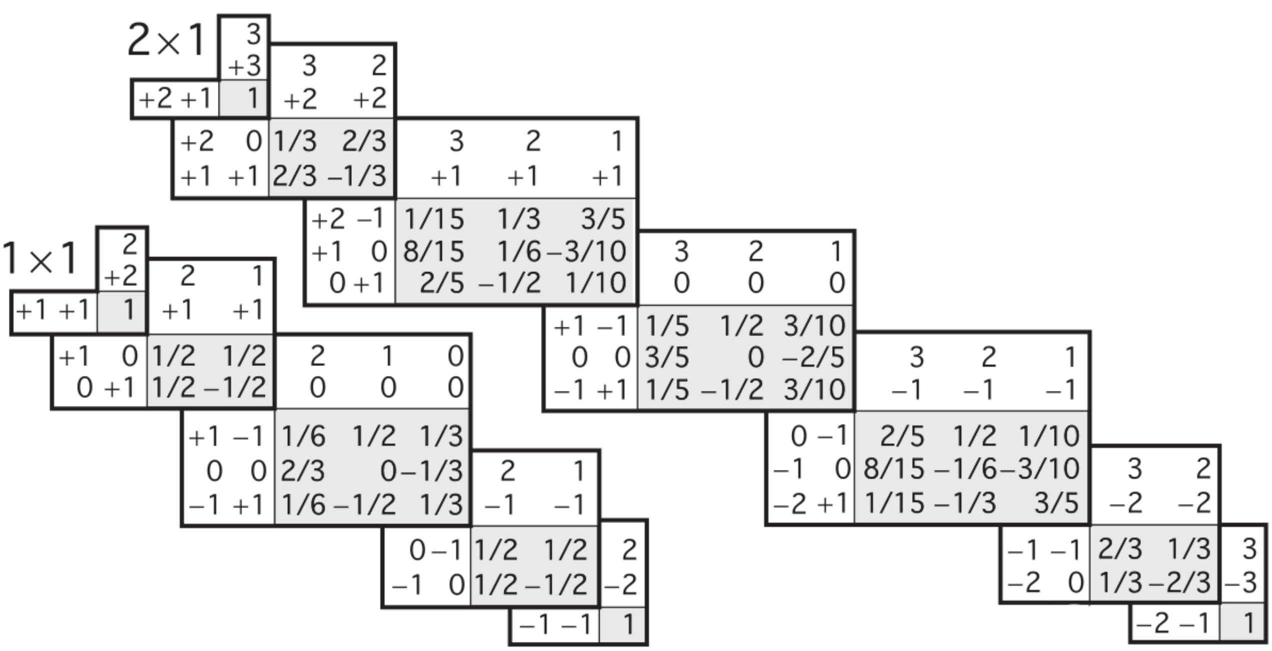
Course Code: PHY4154 Nuclear and Particle Physics **Total marks: 40**
Date: 29/09/2025 **Time: 10.00 AM to 12.00 PM**

Instructions:

1. All questions are compulsory.
2. Please show all your work.
3. Please no answers in SI units. Use the given conversion factors if needed.

$$1\ \text{eV} = 1.609 \times 10^{-19}\ \text{J}, \quad 1\ \text{eV}/c^2 = 1.79 \times 10^{-36}\ \text{kg}$$
$$1\ \text{eV}/c = 5.36 \times 10^{-28}\ \text{kg-m/s}, \qquad hc = 1239\ \text{eV}\ nm$$

Hadron	**Quark content**	**Mass (MeV/c^2)**	$I(J^{PC})$
π^+, π^0, π^-	$u\bar{d}, u\bar{u}, \bar{u}d$	140, 135, 140	$1(0^{-+})$
ρ^+, ρ^0, ρ^-	$u\bar{d}, u\bar{u}, \bar{u}d$	776	$1(1^{--})$
p, n	uud, udd	938.27, 939.57	$\frac{1}{2}\left(\frac{1}{2}^+\right)$
Λ^0	uds	1116	$0\left(\frac{1}{2}^+\right)$
$\Sigma^+, \Sigma^0, \Sigma^-$	uus, uds, dds	1189, 1193, 1197	$1\left(\frac{1}{2}^+\right)$
$\Delta^-, \Delta^0, \Delta^+, \Delta^{++}$	ddd, udd, uud, uuu	1232	$\frac{3}{2}\left(\frac{3}{2}^+\right)$
$\Sigma^{*+}, \Sigma^{*0}, \Sigma^{*-}$	uus, uds, dds	1383, 1384, 1387	$1\left(\frac{3}{2}^+\right)$
Lepton		**Mass (MeV/c^2)**	(J^P)
e, μ, τ		0.511,105,1777	$\left(\frac{1}{2}^+\right)$



Questions

1. (12 marks)

   (a) Consider firing a beam of μ^+ at stationary protons. What is the minimum energy required by the incoming μ^+ to produce a Higgs boson? (Assume the proton to be a fundamental particle with no substructure. Ignore questions of how a muon/proton interaction is giving a Higgs boson!)

   (b) Given the calculated energy in part (a), and that the size of the proton is $\sim 10^{-15}$ m, comment on whether it is okay to treat the proton as a fundamental particle in this problem.

2. (12 marks)

   (a) Consider the following decay $\Delta^{++} \rightarrow p + \pi^+$. Draw the Feynman diagrams of this process through strong and weak interactions. Considering just the strong interaction, what will be the possible angular momentum states (i.e ℓ) of the proton and pion system?

   (b) Consider the following decay $\Sigma^{*-} \rightarrow \Lambda^0 + \pi^-$. Draw the Feynman diagrams of this process through strong and weak interactions. Considering just the *weak* interaction, what will be the possible angular momentum states (i.e. ℓ) of the Λ^0 and pion system.

3. (8 marks) The Σ^{*0} decays through the strong interaction. It can decay to
   (i) $\Sigma^+ + \pi^-$, or (ii) $\Sigma^0 + \pi^0$, or (iii) $\Sigma^- + \pi^+$
   Suppose we observed 100 such decays of the Σ^{*0}, how many would we expect to see of each type.

4. (8 marks) Consider the following two decays: $\rho^0 \rightarrow \pi^0\pi^0$ and $\rho^0 \rightarrow \pi^+\pi^-$. Draw the Feynman diagrams of both decays. Examine (and show) if both decays are allowed or is either of them (or both) forbidden due to any symmetry.

Solutions

1. (12 marks)

   (a) Consider firing a beam of μ^+ at stationary protons. What is the minimum energy required by the incoming μ^+ to produce a Higgs boson? (Assume the proton to be a fundamental particle with no substructure. Ignore questions of how a muon/proton interaction is giving a Higgs boson!)

   (b) Given the calculated energy in part (a), and that the size of the proton is $\sim 10^{-15}$ m, comment on whether it is okay to treat the proton as a fundamental particle in this problem.

(a) Let the momentum of the muon be p_μ. The proton is at rest. After the collision, the Higgs boson will be produced. Conservation of momentum tells us that the momentum of the Higgs boson $p_H = p_\mu = p$ (say).

The total initial energy is $E_i = E_\mu + E_p = \sqrt{p_\mu^2 c^2 + m_\mu^2 c^4} + m_p c^2$

The total final energy is $E_f = \sqrt{p_H^2 c^2 + m_H^2 c^4}$

$$\begin{aligned}
E_i &= E_f \\
\sqrt{p_\mu^2 c^2 + m_\mu^2 c^4} + m_p c^2 &= \sqrt{p_H^2 c^2 + m_H^2 c^4} \\
p_\mu^2 c^2 + m_\mu^2 c^4 + 2 m_p c^2 \sqrt{p_\mu^2 c^2 + m_\mu^2 c^4} &= p_H^2 c^2 + m_H^2 c^4 \\
m_\mu^2 c^4 + 2 m_p c^2 \sqrt{p_\mu^2 c^2 + m_\mu^2 c^4} &= m_H^2 c^4 \\
(0.105)^2 + (0.938)^2 + 2 \times 0.938 \times \sqrt{p^2 c^2 + (0.105)^2} &= (125)^2 \\
\sqrt{p^2 c^2 + (0.105)^2} &= 8329 \\
p^2 c^2 &= (8329)^2 - (0.105)^2 \\
p &\approx 8329 \mathrm{GeV/c}
\end{aligned}$$

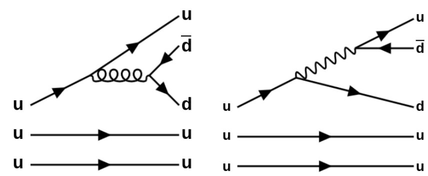
The energy of the muon is $E = \sqrt{p_\mu^2 c^2 + m_\mu^2 c^4} \approx 8329$ GeV.

(b) Considering de Broglie wavelength of these muons,

$$\lambda = \frac{h}{p} = \frac{hc}{pc} = \frac{1239 eV \times 10^{-9} m}{8329 \times 10^9 eV} = 1.5 \times 10^{-19} m$$

Thus the muons will probe a length scale of 10^{-19} m, which is much smaller than the proton. So one cannot treat the protons as fundamental particle in this problem.

2. (a) Consider the following decay $\Delta^{++} \to p + \pi^+$. Draw the Feynman diagrams of this process through strong and weak interactions. Considering just the strong interaction, what will be the possible angular momentum states (i.e ℓ) of the proton and pion system?



Consider the strong decay. Parity is conserved in strong decays.

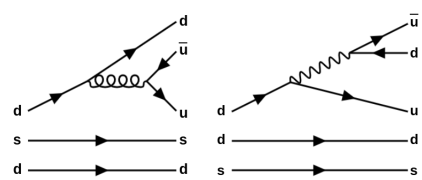
Initial parity is $P_i = P_{\Delta^{++}} = +1$.

Final parity is $P_f = P_p \times P_\pi \times (-1)^\ell$. Since parity is conserved, we get $+1 = (+1)(-1) \times (-1)^\ell$. This says that ℓ has to be $1, 3, ...$

Initial spin is $J_i = J_{\Delta^{++}} = \frac{3}{2}$. Final spin is $J_f = \frac{1}{2} \otimes 0 \otimes \ell$ where ℓ is the angular momentum of the final state.
To get $\frac{3}{2}$, ℓ needs to be 1 or 2.

Only $\ell = 1$ preserves parity. Thus answer is $\ell = 1$.

(b) Consider the following decay $\Sigma^{*-} \to \Lambda^0 + \pi^-$. Draw the Feynman diagrams of this process through strong and weak interactions. Considering just the *weak* interaction, what will be the possible angular momentum states (i.e. ℓ) of the Λ^0 and pion system.



Consider the weak decay. Parity is violated maximally in weak decays.

Initial parity is $P_i = P_{\Sigma^{*-}} = +1$.

Final parity is $P_f = P_{\Lambda^0} \times P_\pi \times (-1)^\ell$. Since parity is violated, we get $-1 = (+1)(-1) \times (-1)^\ell$. This says that ℓ has to be $0, 2, ...$

Initial spin is $J_i = J_{\Sigma^{*-}} = \frac{3}{2}$. Final spin is $J_f = \frac{1}{2} \otimes 0 \otimes \ell$ where ℓ is the angular momentum of the final state.
To get $\frac{3}{2}$, ℓ needs to be 1 or 2.

Only $\ell = 2$ violates parity. Thus answer is $\ell = 2$.

3. (8 marks) The Σ^{*0} decays through the strong interaction. It can decay to
(i) $\Sigma^+ + \pi^-$, or (ii) $\Sigma^0 + \pi^0$, or (iii) $\Sigma^- + \pi^+$
Suppose we observed 100 such decays of the Σ^{*0}, how many would we expect to see of each type.

The decay is strong, so the isospin states will play a role in these decays.
The isospin of the initial state $I_i = I_{\Sigma^{*0}} = |1, 0\rangle$

The final products are the Σ and π, which have $I_\Sigma = |1, I_3\rangle$ and $I_\pi = |1, I_3\rangle$. Thus for the final isospin, we have $I_f = 1 \otimes 1 = 2, 1, 0$.

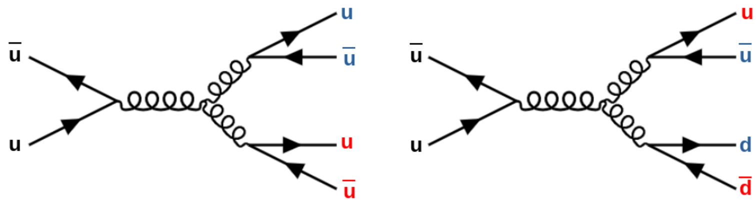
$$\begin{aligned} |1,0\rangle &= a|1,1\rangle|1,-1\rangle + b|1,0\rangle|1,0\rangle + c|1,-1\rangle|1,-1\rangle \\ \Sigma^{*0} \rightarrow &\qquad \Sigma^+ \ \pi^- \qquad\quad \Sigma^0 \ \pi^0 \qquad\quad \Sigma^- \pi^+ \end{aligned}$$

Looking up the Clebsch-Gordon coefficients, we get $a = c = \frac{1}{\sqrt{2}}$ and $b = 0$.

The rates are proportional to $\mathcal{M}^2$, so we see that the
$\Sigma^+ + \pi^-$ and $\Sigma^- + \pi^+$ decays will be $\frac{1}{2}$ times each (so 50 decays each),
and the $\Sigma^0 + \pi^0$ decay will be absent.

4. (8 marks) Consider the following two decays: $\rho^0 \rightarrow \pi^0\pi^0$ and $\rho^0 \rightarrow \pi^+\pi^-$. Draw the Feynman diagrams of both decays. Examine (and show) if both decays are allowed or is either of them (or both) forbidden due to any symmetry.

These decays are strong. The Feynman diagrams are



Consider the decay of $\rho^0 \rightarrow \pi^+\pi^-$. Since its a strong decay, let us check if parity is conserved.
$P_i = P_\rho = (-1)$
$P_f = P_\pi \times P_\pi \times (-1)^\ell = (-1)(-1)(-1)^\ell = (-1)^\ell$

Conserving spin tells us that
$J_i = J_\rho = 1$
$J_f = J_\pi \otimes J_\pi \otimes \ell = 0 \otimes 0 \otimes \ell = J_i$
Thus $\ell = 1$.

$\ell = 1$ gives us that parity is conserved and this decay is allowed.

Now consider the decay of $\rho^0 \rightarrow \pi^0\pi^0$. This is also a strong decay, and thus the argument proceeds the same way to get $\ell = 1$.

However, here $\ell = 1$ is forbidden. The final state consists of two identical bosons, which means the wavefunction has to be symmetric under identical particle exchange. This requires $\ell = 0, 2, ...$

These two requirements on ℓ are at odds.. and thus this decay is forbidden.