PHY 4154

Nuclear and Particle Physics

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Some group theory terms

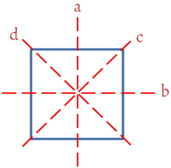
I want to introduce terminology of group theory that you will encounter here. We will certainly not be comprehensive!
A collection of elements satisfying *Closure*, *Identity*, *Inverse*, and *Associativity* is a group, (with a product rule defined).

If U_i are members of a group, then

- Closure: $U_1 U_2 = U_3$ (i.e U_3 is a member of the group)
- Identity: $IU_i = U_i$
- Inverse: $U_i U_i^{-1} = U_i^{-1} U_i = I$ (i.e. U_i^{-1} is member of the group)
- Associativity: $U_i(U_j U_k) = (U_i U_j) U_k$.

For example, the set of all unitary $n \times n$ matrices forms a group, called $U(n)$. Other examples, the set of integers with addition playing the role of *product*.

Symmetry Group of a Square



Elements of group will be
Rotation by 90° clockwise (R_+), and anticlockwise (R_-)
Rotation by 180° (R_{180})
Flip about axes a (R_a), b (R_b), c (R_c), and d (R_d),
and the identity element I (i.e. do nothing).

If the elements commute ($R_i R_j = R_j R_i$), then the group is Abelian.

Is this group Abelian?

What is $R_+ R_a$? What is $R_a R_+$?

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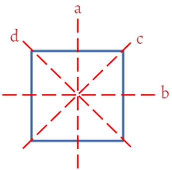

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No, since $R_+ R_a = R_d$, but $R_a R_+ = R_c$.

Rotations

For finite groups: the smallest set of elements whose powers and products generate all elements of group are called the *generators of the group*.
Set of all U responsible for rotations forms a group. As we have seen, J_z is the generator of rotations around z-axis for infinitesimal rotations.

$$U_z(\delta\theta) = e^{-i\delta\theta J_z} = (1 - i\delta\theta J_z)$$

In three dimensions, there are three generators, and they obey $[J_i, J_j] = i\epsilon_{ijk} J_k$

Here, the ϵ_{ijk} are called the structure constants of the group. $\epsilon_{123} = \epsilon_{231} = \epsilon_{312} = 1$ (Others are zero).

Since the $J's$ don't commute, only eigenvalues of one generator (say J_z) are useful quantum numbers.
Non-linear functions of generators that commute with the generators are called Casimir operators. Here $J^2 = J_x^2 + J_y^2 + J_z^2$ is the only Casimir operator, with $[J^2, J_i] = 0$.

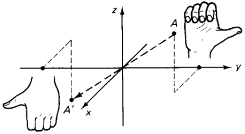
Discrete symmetries : Parity

Consider a "mirroring" or inversion operation

$P\psi(\vec{x}) = \psi(-\vec{x})$

Evidently, $P^2\psi(\vec{x}) = \psi(\vec{x})$.
The eigenvalues of P are thus ± 1.
For the spherical harmonics: $PY_{\ell,m}(\theta,\phi) = (-1)^\ell Y_{\ell,m}(\theta,\phi)$

Parity operation is like mirroring.



Are the laws of physics invariant in a mirror world?

Parity

Typically, $P(\vec{a}) = -\vec{a}$.

But think of $\vec{c} = \vec{a} \times \vec{b}$, here $P(\vec{c}) = \vec{c}$

There are two kinds of vectors, polar vectors (change sign under parity) and axial (or pseudo-) vectors (which do not change sign under parity.

Two kinds of scalars too!
$\vec{a} \cdot \vec{b}$ won't change sign, but $\vec{a} \cdot (\vec{b} \times \vec{c})$ will change sign.

Scalar	$P(s) = s$
Pseudoscalar	$P(p) = -p$
Vector (polar vector)	$P(\vec{v}) = -\vec{v}$
Pseudovector (axial)	$P(\vec{a}) = \vec{a}$

Scalars and pseudovectors have eigenvalue $+1$ for Parity, while pseudoscalars and vectors have eigenvalue -1.

Parity

We assign parity to all particles.
eg. Quarks have positive parity, antiquarks are negative.

Parity is a multiplicative quantum number. The parity of a composite particle is the product of the parities of its constituents.

Consider a reaction $a + b \to c + d$
$|\mathrm{initial}\rangle = |a\rangle|b\rangle|\mathrm{Relative\ motion}\rangle$,
thus parity is $P(|\mathrm{initial}\rangle) = P(|a\rangle)\ P(|b\rangle)\ P(|\mathrm{motion}\rangle)$

Parity for mesons has additional factor of $(-1)^{\ell}$.

Names of particles tell us parity, for example pions are pseudoscalar mesons (i.e. spin is 0 and parity is -1). A photon is a vector particle (i.e. spin is 1 and parity is -1).

$\tau - \theta$ Puzzle

Two particles (τ and θ), with same mass, spin, charge, however decayed into states of opposite parity

$$\begin{aligned} \theta^+ &\to \pi^+\pi^0 & \qquad (P = (-1)^2 = +1) \\ \tau^+ &\to \pi^+\pi^0\pi^0 & \qquad (P = (-1)^3 = -1) \\ &\to \pi^+\pi^+\pi^- & \end{aligned}$$

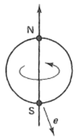
Turns out, τ and θ are the same particle, the K^+, and its just that parity is not conserved.
This $\tau - \theta$ puzzle is what led to the hunt for parity violation, and eventually Prof. Wu's experiment.

Parity is conserved in electromagnetic and strong interactions, not in weak interactions.

Prof. Wu and $^{60}\mathrm{Co}$

Prof. Wu's famous experiment. Consider the beta decay of Cobalt 60,
$^{60}\mathrm{Co} \rightarrow ^{60}\mathrm{Ni} + e^- + \bar{\nu}_e$

Take a collection of $^{60}\mathrm{Co}$ nuclei, and align them such that their spins point in $+z$ direction. Then measure the direction of the emitted electrons



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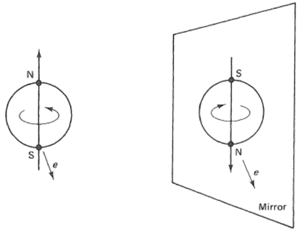


In regular world, electrons come out anti-aligned to nuclear spin. In the mirror world, electrons come out aligned to nuclear spin.

This is profound! An unambiguous way of differentiating left from right.

Helicity

By convention, we take m to be along the $z - axis$. Let us consider defining the axis by the direction of motion of the particle.

(a) Right-handed (b) Left-handed

We define helicity $\equiv m_s/s$

A particle of spin ½ can have helicity of $+1$ (right-handed) or -1 (left-handed).

Helicity however is not Lorentz-invariant.
Consider a right-handed particle, and imagine a frame traveling to the right at speed greater than v. Then the particle would be going to left, and helicity would be different.

Helicity of neutrinos

One place where the previous argument breaks down. Consider (massless) neutrinos.

Its not possible to go faster than them, and thus helicity of a neutrino (or any massless) particle is Lorentz-invariant.

Remarkably,

$$\text{Neutrinos are left handed}$$
$$\text{Antineutrinos are right handed.}$$

Experimentally

Measuring the spin of muons, and knowing that pions are spin-0, the muon and antineutrino spins are oppositely aligned.

Parity violation

Photons come with helicity $+1$ or -1, equally likely.

Neutrinos are only helicity$=-1$ (left handed).

Mirror-image of a neutrino does not exist!!

Parity violation happens in weak processes, but parity is a symmetry of strong and electromagnetic interactions.

Charged pion decay

$$\pi^- \to \mu^- + \bar{\nu}_\mu \qquad : \quad 99.99\ \%$$

$$\pi^- \to e^- + \bar{\nu}_e \qquad : \quad 1.23 \times 10^{-4}\ \%$$

Why?

Charge conjugation

Charge conjugation changes particle to its antiparticle. The charge conjugation operator is such that it changes the sign of all internal quantum numbers (charge, baryon number, lepton number, strangeness, charm etc.) while leaving mass, energy, momentum, spin unchanged.

$C|p\rangle = |\bar{p}\rangle$, $C|\nu_e\rangle = |\bar{\nu}_e\rangle$, $C|\gamma\rangle = |\gamma\rangle$

$C^2 = I$, similar to Parity. Thus the eigenvalues are ± 1.

Only particles that are their own antiparticles are eigenstates of C. (eg. photon, π^0, ψ ...)

The eigenvalue for the photon is -1, for a π^0 is $+1$.

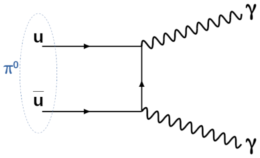
A system that is a state of a spin-½ particle and its antiparticle, will have C eigenstate of $(-1)^{\ell+s}$ (ℓ: angular momentum state, s: spin state).

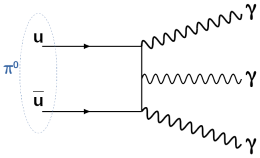
Charge conjugation is also a multiplicative quantum number.

Charge conjugation

Charge conjugation is conserved in strong and electromagnetic interactions (like parity).

Consider neutral pion decay $\pi^0 \to \gamma\gamma$





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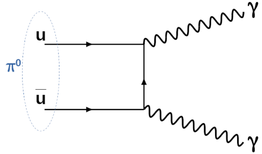

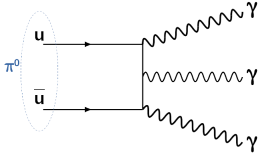

Is the diagram on the right a valid diagram?

Let us apply charge conjugation conservation.

$+1 = -1 \times -1$ $\qquad\qquad$ $+1 \neq -1 \times -1 \times -1$

$\pi^0 \to 2\gamma$ is allowed, but $\pi^0 \to 3\gamma$ is not allowed by charge conjugation conservation.

So far...

What have we learned so far?

Each particle carries a quantum number called parity. Parity (P) is conserved strong and EM interactions, but is maximally violated in weak interactions.

Charge conjugation number is conserved in strong and EM interactions. Very few particles are eigenstates of C.

Parity and Charge conjugation, P and C, are multiplicative quantum numbers.

We write particles/states as J^{PC}.

Remember to understand that the C is only associated for particles that are their own antiparticles.

J^{PC}

Hadron	**Quark content**	**Mass (MeV/c^2)**	$I(J^{PC})$
π^+, π^0, π^-	$u\bar{d}, u\bar{u}, \bar{u}d$	140, 135, 140	$1(0^{-+})$
$K^+, K^-, K^0, \bar{K}^0$	$u\bar{s}, \bar{u}s, d\bar{s}, \bar{d}s$	494, 494, 498,498	$\frac{1}{2}(0^-)$
ρ^+, ρ^0, ρ^-	$u\bar{d}, u\bar{u}, \bar{u}d$	776	$1(1^{--})$
p, n	uud, udd	938.27, 939.57	$\frac{1}{2}\left(\frac{1}{2}^+\right)$
$\Sigma^+, \Sigma^0, \Sigma^-$	uus, uds, dds	1189, 1193, 1197	$1\left(\frac{1}{2}^+\right)$
$\Delta^-, \Delta^0, \Delta^+, \Delta^{++}$	ddd, udd, uud, uuu	1232	$\frac{3}{2}\left(\frac{3}{2}^+\right)$
$\Sigma^{*+}, \Sigma^{*0}, \Sigma^{*-}$	uus, uds, dds	1383, 1384, 1387	$1\left(\frac{3}{2}^+\right)$
Ω^-	sss	1672	$0\left(\frac{3}{2}^+\right)$
Lepton		**Mass (MeV/c^2)**	(J^P)
e, μ, τ		0.511,105,1777	$\left(\frac{1}{2}^+\right)$
ν_e, ν_μ, ν_τ		not known	$\left(\frac{1}{2}^+\right)$

Apply to baryon decays

Problem 1: Consider the strong decay of the Δ^{++} baryon ($\Delta^{++} \to p + \pi^{+}$). Draw the Feynman diagram of this decay. What will be the angular momentum state (value of ℓ) of the proton and the pion in the Δ rest frame

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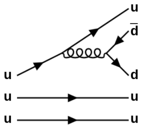
This is a strong decay. Thus parity is expected to be conserved.



$P_{initial} = +1$, and thus P_{final} must also be $+1$.
$P_{final} = P_p \cdot P_\pi \cdot (-1)^\ell = (+1)(-1)(-1)^\ell$

The initial spin (J) of the Δ^{++} is $\frac{3}{2}$. This is has to match the final J.
$J_{final} = \frac{1}{2} \otimes 0 \otimes \ell$.
To get $\frac{3}{2}$, ℓ needs to be 1 or 2. Only $\ell = 1$ preserves parity. Thus the angular momentum state will be $\ell = 1$ for the proton and pion system.

CP

Let's go back and consider the pion decay

$$\pi^- \rightarrow \mu^-_{RH} + \bar{\nu}_{\mu\ RH}$$

If we apply parity operator, reaction doesn't work ($\bar{\nu}_{\mu\ LH}$ doesn't exist).

Consider applying the charge conjugation operator

$$C \longrightarrow \quad \pi^+ \rightarrow \mu^+_{RH} + \nu_{\mu\ RH}$$

This doesn't work since $\nu_{\mu\ RH}$ doesn't exist.

Now consider applying *CP* together (*P* followed by *C*).

$$CP \longrightarrow \quad \pi^+ \rightarrow \mu^+_{LH} + \nu_{\mu\ LH}$$

This works great!

Perhaps *CP* is what meant all along by mirror symmetry. If we do an inversion, followed by replacing each particle by its antiparticle, then we restore our *intuitive* sense. Perhaps *CP* is conserved in weak interactions.

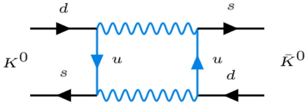
CP and Kaons

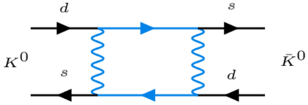
Neutral kaons are pseudoscalars, so we have

$$P|K^0\rangle = -|K^0\rangle \quad P|\overline{K}^0\rangle = -|\overline{K}^0\rangle$$
$$C|K^0\rangle = |\overline{K}^0\rangle \quad C|\overline{K}^0\rangle = |K^0\rangle$$
$$CP|K^0\rangle = -|\overline{K}^0\rangle \quad CP|\overline{K}^0\rangle = -|K^0\rangle$$

These two, K^0 and $\overline{K}^0$ have no distinguishing conserved quantum number. Nothing prohibits their oscillation!

Here are two example diagrams showing how a K^0 can turn into a $\overline{K}^0$





What we observe will be a linear combination of K^0 and $\overline{K}^0$.

$|K_1\rangle = (\frac{1}{\sqrt{2}})(|K^0\rangle - |\overline{K}^0\rangle)$ and $|K_2\rangle = (\frac{1}{\sqrt{2}})(|K^0\rangle + |\overline{K}^0\rangle)$

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$CP|K_1\rangle = |K_1\rangle$ and $CP|K_2\rangle = -|K_2\rangle$

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What we observe will be K_1 and K_2. These are also eigenstates of CP

$CP|K_1\rangle = |K_1\rangle$ and $CP|K_2\rangle = -|K_2\rangle$

Now if CP is conserved in weak interactions, then K_1 must decay to a state with $CP = +1$ and K_2 to a state with $CP = -1$.

It turns out that the allowed decays are only
$|K_1\rangle \rightarrow 2\pi$ and $|K_2\rangle \rightarrow 3\pi$

Moreover the 2π decay is faster by a factor of 100 than then 3π decay.

CP and Kaons

$|K_1\rangle = (\frac{1}{\sqrt{2}})(|K^0\rangle - |\overline{K}^0\rangle)$ and $|K_2\rangle = (\frac{1}{\sqrt{2}})(|K^0\rangle + |\overline{K}^0\rangle)$

So $|K^0\rangle = (\frac{1}{\sqrt{2}})(|K_1\rangle + |K_2\rangle)$ and $|\overline{K}^0\rangle = (\frac{1}{\sqrt{2}})(|K_2\rangle - |K_1\rangle)$

If we start with a beam of $|K^0\rangle$, then we will first lose the K_1's as they decay to 2π, and we shall be left with a beam of pure K_2's. Near the source we shall see lots of 2π events, but farther down only the 3π events.

Thus CP invariance in weak interactions says that considering the neutral pion system, there are short-lived neutral K mesons (K_1), and long-lived neutral K mesons (K_2).

In 1956, Lederman and group found lifetimes to be

$$\tau_1 = 0.895 \times 10^{-10}\ \mathrm{s}$$
$$\tau_2 = 5.11 \times 10^{-8}\ \mathrm{s}$$

with a negligible difference in mass ($m_2 - m_1 = 3.48 \times 10^{-6}\ \mathrm{eV}/c^2$).

So is K^0 the "real" particle or K_1?

CP violation

CP eigenstates are $|K_1\rangle$ and $|K_2\rangle$

Since K_1 decays rapidly, let us call is $K^0_{\rm S}$ (i.e. K^0-short) and let us call K_2 as $K^0_{\rm L}$ (i.e. K^0-long).

Now, at all "late times" we expect to have only $K^0_{\rm L}$, and thus have only 3π decays.

But of course... experimentally we do see some 2π decays at the end of a beam (at a late time!).

Thus, $|K^0_{\rm L}\rangle \neq |K_2\rangle$, but rather $|K^0_{\rm L}\rangle = \dfrac{1}{\sqrt{1+|\epsilon|^2}}(|K_2\rangle + \epsilon|K_1\rangle)$

The long-lived neutral kaon is *not* a perfect eigenstate of *CP*, but contains a little bit of an admixture.

Experimentally, $\epsilon = 2.24 \times 10^{-3}$, and its non-zero value is a measure of nature's departure from perfect CP invariance.

CP violation

There is another way that K^0_{L} can decay

$$K^0_{\mathrm{L}} \to \pi^+ + e^- + \bar{\nu}_e$$
$$K^0_{\mathrm{L}} \to \pi^- + e^+ + \nu_e$$

CP takes one to the other, so if CP is conserved, then both of these decays should be equal.

CP violation

There is another way that K_{L}^0 can decay

$$K_{\mathrm{L}}^0 \to \pi^+ + e^- + \bar{\nu}_e$$
$$K_{\mathrm{L}}^0 \to \pi^- + e^+ + \nu_e$$

CP takes one to the other, so if CP is conserved, then both of these decays should be equal.

However, the positron decay is more common by 0.33%

Clear distinction between matter and antimatter.
This CP violation is now observed in several other decays such as neutral B mesons (and recently in D mesons). We have several sources for CP violation in standard model, however they are insufficient to explain the observed matter-antimatter asymmetry in the universe...