PHY 4154

Nuclear and Particle Physics

Sourabh Dube

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Nucleon Isospin

- Heisenberg noticed the similarities in masses between neutron and proton. ($m_p = 938.28 \ \ \mathrm{MeV}/c^2$, $m_n = 939.57 \ \ \mathrm{MeV}/c^2$).
- "Their charge is different, but aside from this they are identical"
- Charge independence of hadronic force.
- What gives rise to this symmetry?

Nucleon Isospin

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In analogy to spin, nucleon has $I = \frac{1}{2}$, and we have two possible states

$$|\frac{1}{2}, \frac{1}{2}\rangle = \text{ proton } ; \quad |\frac{1}{2}, -\frac{1}{2}\rangle = \text{ neutron}$$

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Note that components are not in regular space, but in abstract 'isospin' space.

Nucleon Isospin

The generators of isospin are $I_i = \frac{1}{2}\tau_i$

$$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Recall the Pauli spin matrices, with $\hat{\mathbf{S}} = \left(\frac{\hbar}{2}\right)\sigma$

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

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However we know $[H_{had} + H_{em}, Q] = 0$.

We could define charge for $|I, I_3\rangle$ as $q = e(I_3 + \frac{1}{2})$.

This gives $q_p = 1$ ($I_3 = ½$), and $q_n = 0$ ($I_3 = -½$).

We have $[H_{had} + H_{em}, I_3] = 0$.

Pion Isospin

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Isospin is conserved, and nucleon isospin $I = ½$. What possible values can pion isospin have?

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$$\frac{1}{2} \to \frac{1}{2} \oplus I_\pi$$

It can be 0 or 1 .
If $I = 0$, there would be one pion. But there are three pions $(\pi^\pm, \pi^0)$.
Thus for pions we assign $I = 1$.

$$\pi^+ = |1, 1\rangle, \quad \pi^0 = |1, 0\rangle, \quad \pi^- = |1, -1\rangle$$

Δ Isospin

For Δ's, $I = \frac{3}{2}$

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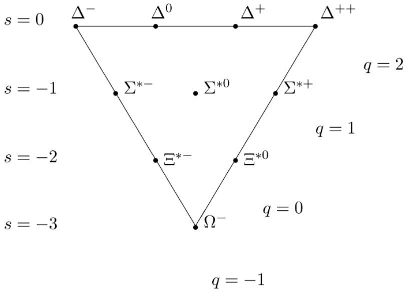

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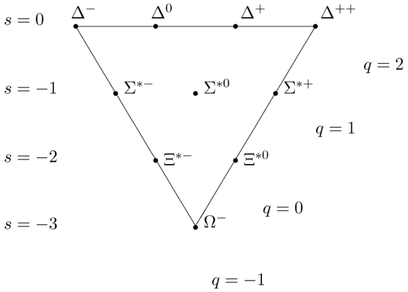

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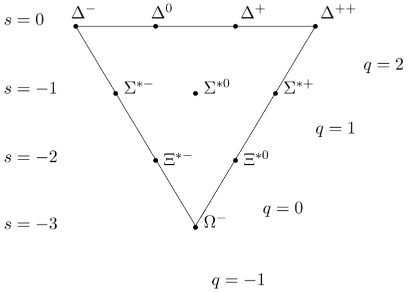

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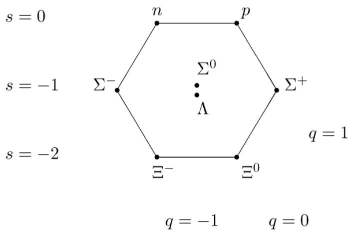
For hadrons composed of only the u, d, s quarks, we have the Gell-Mann–Nishijima formula $Q = I_3 + ½(B + S)$. B is baryon number, and S is strangeness.

Isospin



$Q = I_3 + \frac{1}{2}(B + S)$, what is Σ and Ξ isospins?

Σ^+	uus	Σ^0	uds
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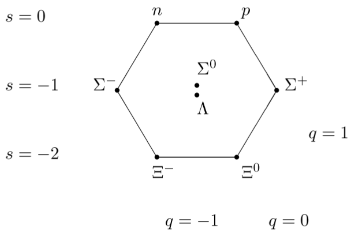

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In context of quark model, it can follow from u, d being a isospin doublet ($I = ½$), and s having isospin zero. (All other flavors also get isospin zero, as do leptons and mediators).

Implications of Isospin

Consider adding the isospins of p and n, i.e. a state with two nucleons (each with $I = ½$), will result in

$$
\begin{aligned}
|1,1\rangle &= pp \\
|1,0\rangle &= \left(\frac{1}{\sqrt{2}}\right)(pn + np) \\
|1,-1\rangle &= nn \\
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Experimentally we only see one bound state of p and n (and no pp or nn bound states). Thus deuteron must be isosinglet (strong attraction in $I = 0$ channel, not in $I = 1$ channel). The deuteron isospin is thus 0.

Nucleon nucleon scattering

Consider the following processes

(a) $p + p \to d + \pi^+$

(b) $p + n \to d + \pi^0$

(c) $n + n \to d + \pi^-$

Deuteron has $I = 0$.
On RHS isospin states are defined by pion isospin:
$|1, 1\rangle$, $|1, 0\rangle$, $|1, -1\rangle$.

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On LHS (see prev slide), the *pp* and *nn* states are $|1, 1\rangle$, and $|1, -1\rangle$.
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Since isospin is conserved, only the $I = 1$ state will contribute. Thus the scattering amplitudes are in the ratio $\mathcal{M}_a : \mathcal{M}_b : \mathcal{M}_c = 1 : \sqrt{\frac{1}{2}} : 1$.

The cross section, σ, goes like $|\mathcal{M}|^2$, and thus are in the ratio $\sigma_a : \sigma_b : \sigma_c = 2 : 1 : 2$.

Cross section σ is the rate at which a process occurs, and is measured in units of area (more on this later).

Pion-nucleon scattering

Pion-nucleon scattering processes ($\pi N \to \pi N$).

Elastic processes

(a) $\pi^+ + p \to \pi^+ + p$

(b) $\pi^0 + p \to \pi^0 + p$

(c) $\pi^- + p \to \pi^- + p$

(d) $\pi^+ + n \to \pi^+ + n$

(e) $\pi^0 + n \to \pi^0 + n$

(f) $\pi^- + n \to \pi^- + n$

Charge-exchange processes

(g) $\pi^+ + n \to \pi^0 + p$

(h) $\pi^0 + p \to \pi^+ + n$

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The pion has $I = 1$, and the nucleon has $I = ½$
thus the total isospin can be $3/2$ or $1/2$.
There are only two distinct scattering amplitudes here
(strong processes invariant under rotations in isospin space),
lets call these as $\mathcal{M}_3$ for $I = \frac{3}{2}$ and $\mathcal{M}_1$ for $I = ½$.

Pion-nucleon scattering

$$\pi^+ + p : |1,1\rangle|\tfrac{1}{2},\tfrac{1}{2}\rangle = |\tfrac{3}{2},\tfrac{3}{2}\rangle$$

$$\pi^0 + p : |1,0\rangle|\tfrac{1}{2},\tfrac{1}{2}\rangle = \sqrt{\tfrac{2}{3}}\,|\tfrac{3}{2},\tfrac{1}{2}\rangle - \sqrt{\tfrac{1}{3}}\,|\tfrac{1}{2},\tfrac{1}{2}\rangle$$

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(f)	$\pi^- + n \to \pi^- + n$	$\left\|\frac{3}{2},-\frac{3}{2}\right\rangle \to \left\|\frac{3}{2},-\frac{3}{2}\right\rangle$
(c)	$\pi^- + p \to \pi^- + p$	$\sqrt{\frac{1}{3}}\left\|\frac{3}{2},-\frac{1}{2}\right\rangle - \sqrt{\frac{2}{3}}\left\|\frac{1}{2},-\frac{1}{2}\right\rangle \to \sqrt{\frac{1}{3}}\left\|\frac{3}{2},-\frac{1}{2}\right\rangle - \sqrt{\frac{2}{3}}\left\|\frac{1}{2},-\frac{1}{2}\right\rangle$
(b)	$\pi^0 + p \to \pi^0 + p$	$\sqrt{\frac{2}{3}}\left\|\frac{3}{2},\frac{1}{2}\right\rangle - \sqrt{\frac{1}{3}}\left\|\frac{1}{2},\frac{1}{2}\right\rangle \to \sqrt{\frac{2}{3}}\left\|\frac{3}{2},\frac{1}{2}\right\rangle - \sqrt{\frac{1}{3}}\left\|\frac{1}{2},\frac{1}{2}\right\rangle$
(j)	$\pi^- + p \to \pi^0 + n$	$\sqrt{\frac{1}{3}}\left\|\frac{3}{2},-\frac{1}{2}\right\rangle - \sqrt{\frac{2}{3}}\left\|\frac{1}{2},-\frac{1}{2}\right\rangle \to \sqrt{\frac{2}{3}}\left\|\frac{3}{2},-\frac{1}{2}\right\rangle + \sqrt{\frac{1}{3}}\left\|\frac{1}{2},-\frac{1}{2}\right\rangle$

We have $\mathcal{M}_a = \mathcal{M}_f = \mathcal{M}_3$

$\mathcal{M}_c = \frac{1}{3}\mathcal{M}_3 + \frac{2}{3}\mathcal{M}_1$

$\mathcal{M}_b = \frac{2}{3}\mathcal{M}_3 + \frac{1}{3}\mathcal{M}_1$

$\mathcal{M}_j = \frac{\sqrt{2}}{3}\mathcal{M}_3 - \frac{\sqrt{2}}{3}\mathcal{M}_1$

$\sigma_a : \sigma_c : \sigma_j = 9|\mathcal{M}_3|^2 \colon |\mathcal{M}_3 + 2\mathcal{M}_1|^2 \colon 2|\mathcal{M}_3 - \mathcal{M}_1|^2$

Quick data analysis concept

Consider doing this scattering experiment (with $\pi^\pm$ and protons).

Let me explain one quick concept first.

Suppose you have a decay of particle $A \to B + C$

Evidently conservation of energy-momentum $\Rightarrow p_A = p_B + p_C$
(where p are 4-vectors!)

Experimentally we measure p_B and p_C, and thus calculate $(p_B + p_C)^2$,
but this is just p_A^2 and $p_A^2 = m_A^2$
i.e. $\sqrt{(p_B + p_C)^2} = m_A$

Thus by measuring the 4-vectors of the daughters (B, C), we are able to infer the mass of the mother (A). The LHS calculated quantity is called the invariant mass of B and C and is calculated from the observed/measured 4-vectors of B and C.

Invariant mass

Given two 4-vectors p and q, with components (E_p, p_x, p_y, p_z) and (E_q, q_x, q_y, q_z) respectively,

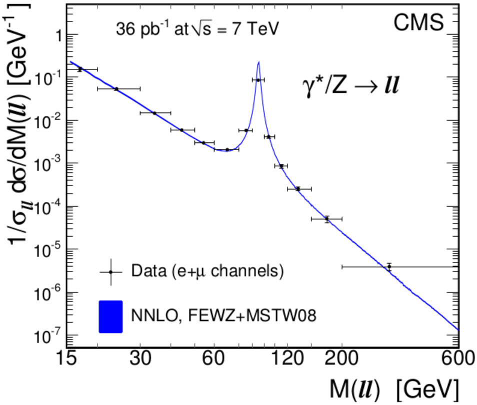
the invariant mass is defined by first adding the two vectors

$$r = p + q = (E_p + E_q, p_x + q_x, p_y + q_y, p_z + q_z)$$

and then calculating

$$r^2 = \left[(E_p + E_q)^2 - (p_x + q_x)^2 - (p_y + q_y)^2 - (p_z + q_z)^2\right]$$

Invariant mass example



The invariant mass spectrum of two charged leptons.
i.e either two muons ($\mu^+\mu^-$) or two electrons (e^+e^-)

Pion-nucleon scattering

Consider doing this scattering experiment (with $\pi^{\pm}$ and protons). Given that measuring total cross section is easier, we can measure the total cross section of $(\pi^{+} + p)$ vs $(\pi^{-} + p)$

(a) $\pi^{+} + p \rightarrow \pi^{+} + p$

(c) $\pi^{-} + p \rightarrow \pi^{-} + p$

(j) $\pi^{-} + p \rightarrow \pi^{0} + n$

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At a CM energy of 1232 MeV, we get the Δ 'resonance' ($I = \frac{3}{2}$), thus $\mathcal{M}_3 \gg \mathcal{M}_1$ and

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$$\frac{\sigma_{tot}(\pi^+ + p)}{\sigma_{tot}(\pi^- + p)} = 3$$

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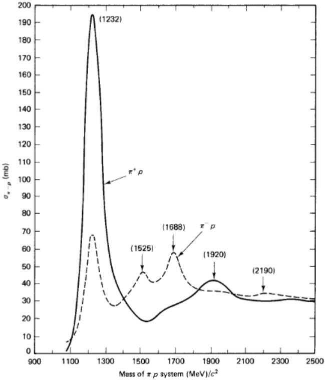

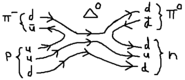

Spectrum from Griffiths, quark flow diagram from A.J.Barr