

PHY 4154

Nuclear and Particle Physics

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Aug 2025

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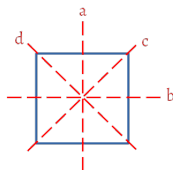
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For example, the set of all unitary $n \times n$ matrices forms a group, called $U(n)$. Other examples, the set of integers with addition playing the role of *product*.

Symmetry Group of a Square



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Elements of group will be

Rotation by 90° clockwise (R_+), and anticlockwise (R_-)

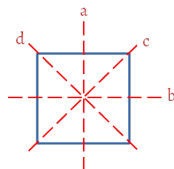
Rotation by 180° (R_{180})

Flip about axes a (R_a), b (R_b), c (R_c), and d (R_d),
and the identity element I (i.e. do nothing).

If the elements commute ($R_i R_j = R_j R_i$), then the group is Abelian.

Is this group Abelian?

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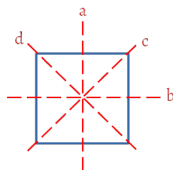
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No, since $R_+ R_a = R_d$, but $R_a R_+ = R_c$.

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Non-linear functions of generators that commute with the generators are called Casimir operators. Here $J^2 = J_x^2 + J_y^2 + J_z^2$ is the only Casimir operator, with $[J^2, J_i] = 0$.

Discrete symmetries : Parity

Consider a “mirroring” or inversion operation

$$P\psi(\vec{x}) = \psi(-\vec{x})$$

Evidently, $P^2\psi(\vec{x}) = \psi(\vec{x})$.

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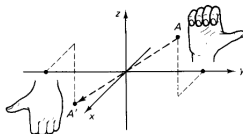
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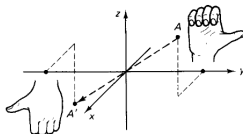
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Are the laws of physics invariant in a mirror world?

Parity

Typically, $P(\vec{a}) = -\vec{a}$.

But think of $\vec{c} = \vec{a} \times \vec{b}$, here $P(\vec{c}) = \vec{c}$

There are two kinds of vectors, polar vectors (change sign under parity) and axial (or pseudo-) vectors (which do not change sign under parity).

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Scalar	$P(s) = s$
Pseudoscalar	$P(p) = -p$
Vector (polar vector)	$P(\vec{v}) = -\vec{v}$
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Scalars and pseudovectors have eigenvalue $+1$ for Parity, while pseudoscalars and vectors have eigenvalue -1 .

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Parity is a multiplicative quantum number. The parity of a composite particle is the product of the parities of its constituents.

Consider a reaction $a + b \rightarrow c + d$

$|\text{initial}\rangle = |a\rangle|b\rangle|\text{Relative motion}\rangle,$

thus parity is $P(|\text{initial}\rangle) = P(|a\rangle) P(|b\rangle) P(|\text{motion}\rangle)$

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Names of particles tell us parity, for example pions are pseudoscalar mesons (i.e. spin is 0 and parity is -1). A photon is a vector particle (i.e. spin is 1 and parity is -1).

$\tau - \theta$ Puzzle

Two particles (τ and θ), with same mass, spin, charge, however decayed into states of opposite parity

$$\theta^+ \rightarrow \pi^+ \pi^0 \qquad (P = (-1)^2 = +1)$$

$$\tau^+ \rightarrow \pi^+ \pi^0 \pi^0 \qquad (P = (-1)^3 = -1)$$

$$\rightarrow \pi^+ \pi^+ \pi^-$$

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Turns out, τ and θ are the same particle, the K^+ , and its just that parity is not conserved.

This $\tau - \theta$ puzzle is what led to the hunt for parity violation, and eventually Prof. Wu's experiment.

Parity is conserved in electromagnetic and strong interactions, not in weak interactions.

Prof. Wu and ^{60}Co

Prof. Wu's famous experiment. Consider the beta decay of Cobalt 60,
 $^{60}\text{Co} \rightarrow ^{60}\text{Ni} + e^- + \bar{\nu}_e$

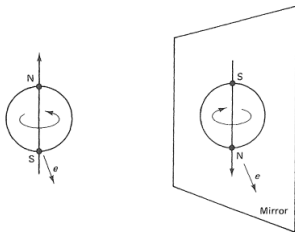
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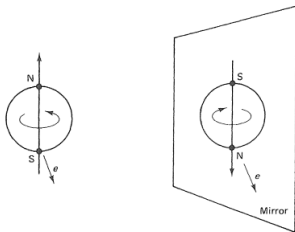
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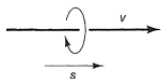


In regular world, electrons come out anti-aligned to nuclear spin. In the mirror world, electrons come out aligned to nuclear spin.

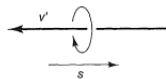
This is profound! An unambiguous way of differentiating left from right.

Helicity

By convention, we take m to be along the z - axis. Let us consider defining the axis by the direction of motion of the particle.



(a) Right-handed



(b) Left-handed

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Helicity however is not Lorentz-invariant.

Consider a right-handed particle, and imagine a frame traveling to the right at speed greater than v . Then the particle would be going to left, and helicity would be different.

Helicity of neutrinos

One place where the previous argument breaks down. Consider (massless) neutrinos.

Its not possible to go faster than them, and thus helicity of a neutrino (or any massless) particle is Lorentz-invariant.

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Experimentally



Measuring the spin of muons, and knowing that pions are spin-0, the muon and antineutrino spins are oppositely aligned.

Parity violation

Photons come with helicity $+1$ or -1 , equally likely.

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Parity violation happens in weak processes, but parity is a symmetry of strong and electromagnetic interactions.

Charged pion decay



$$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu \quad : \quad 99.99 \%$$

$$\pi^- \rightarrow e^- + \bar{\nu}_e \quad : \quad 1.23 \times 10^{-4} \%$$

Why?

Charge conjugation

Charge conjugation changes particle to its antiparticle. The charge conjugation operator is such that it changes the sign of all internal quantum numbers (charge, baryon number, lepton number, strangeness, charm etc.) while leaving mass, energy, momentum, spin unchanged.

$$C|p\rangle = |\bar{p}\rangle, C|\nu_e\rangle = |\bar{\nu}_e\rangle, C|\gamma\rangle = |\gamma\rangle$$

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$C^2 = I$, similar to Parity. Thus the eigenvalues are ± 1 .

Only particles that are their own antiparticles are eigenstates of C . (eg. photon, π^0 , ψ ...)

The eigenvalue for the photon is -1 , for a π^0 is $+1$.

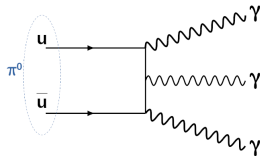
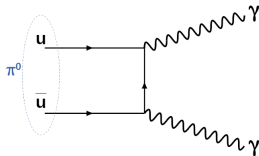
A system that is a state of a spin- $\frac{1}{2}$ particle and its antiparticle, will have C eigenstate of $(-1)^{\ell+s}$ (ℓ : angular momentum state, s : spin state).

Charge conjugation is also a multiplicative quantum number.

Charge conjugation

Charge conjugation is conserved in strong and electromagnetic interactions (like parity).

Consider neutral pion decay $\pi^0 \rightarrow \gamma\gamma$

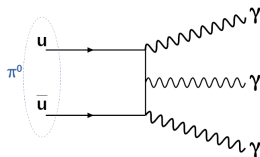
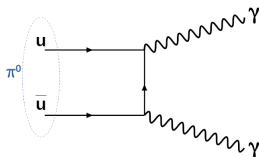


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Is the diagram on the right a valid diagram?

Let us apply charge conjugation conservation.

$$+1 = -1 \times -1$$

$$+1 \neq -1 \times -1 \times -1$$

$\pi^0 \rightarrow 2\gamma$ is allowed, but $\pi^0 \rightarrow 3\gamma$ is not allowed by charge conjugation conservation.

So far...

What have we learned so far?

Each particle carries a quantum number called parity. Parity (P) is conserved strong and EM interactions, but is maximally violated in weak interactions.

Charge conjugation number is conserved in strong and EM interactions. Very few particles are eigenstates of C .

Parity and Charge conjugation, P and C , are multiplicative quantum numbers.

We write particles/states as J^{PC} .

Remember to understand that the C is only associated for particles that are their own antiparticles.

Hadron	Quark content	Mass (MeV/c ²)	$I(J^{PC})$
π^+, π^0, π^-	$u\bar{d}, u\bar{u}, \bar{u}d$	140, 135, 140	$1(0^{-+})$
$K^+, K^-, K^0, \bar{K}^0$	$u\bar{s}, \bar{u}s, d\bar{s}, \bar{d}s$	494, 494, 498, 498	$\frac{1}{2}(0^-)$
ρ^+, ρ^0, ρ^-	$u\bar{d}, u\bar{u}, \bar{u}d$	776	$1(1^{--})$
p, n	uud, udd	938.27, 939.57	$\frac{1}{2} \left(\frac{1}{2}^+ \right)$
$\Sigma^+, \Sigma^0, \Sigma^-$	uus, uds, dds	1189, 1193, 1197	$1 \left(\frac{1}{2}^+ \right)$
$\Delta^-, \Delta^0, \Delta^+, \Delta^{++}$	ddd, udd, uud, uuu	1232	$\frac{3}{2} \left(\frac{3}{2}^+ \right)$
$\Sigma^{*+}, \Sigma^{*0}, \Sigma^{*-}$	uus, uds, dds	1383, 1384, 1387	$1 \left(\frac{3}{2}^+ \right)$
Ω^-	sss	1672	$0 \left(\frac{3}{2}^+ \right)$
Lepton		Mass (MeV/c ²)	(J^P)
e, μ, τ		0.511, 105, 1777	$\left(\frac{1}{2}^+ \right)$
ν_e, ν_μ, ν_τ		not known	$\left(\frac{1}{2}^+ \right)$

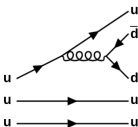
Apply to baryon decays

Problem 1: Consider the strong decay of the Δ^{++} baryon ($\Delta^{++} \rightarrow p + \pi^+$). Draw the Feynman diagram of this decay. What will be the angular momentum state (value of ℓ) of the proton and the pion in the Δ rest frame

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This is a strong decay. Thus parity is expected to be conserved.



$P_{initial} = +1$, and thus P_{final} must also be $+1$.

$$P_{final} = P_p \cdot P_\pi \cdot (-1)^\ell = (+1)(-1)(-1)^\ell$$

The initial spin (J) of the Δ^{++} is $\frac{3}{2}$. This has to match the final J .

$$J_{final} = \frac{1}{2} \otimes 0 \otimes \ell.$$

To get $\frac{3}{2}$, ℓ needs to be 1 or 2. Only $\ell = 1$ preserves parity. Thus the angular momentum state will be $\ell = 1$ for the proton and pion system.

CP

Let's go back and consider the pion decay

$$\pi^- \rightarrow \mu^-_{RH} + \bar{\nu}_{\mu\ RH}$$

If we apply parity operator, reaction doesn't work ($\bar{\nu}_{\mu\ LH}$ doesn't exist).

Consider applying the charge conjugation operator

$$C \longrightarrow \pi^+ \rightarrow \mu^+_{RH} + \nu_{\mu\ RH}$$

This doesn't work since $\nu_{\mu\ RH}$ doesn't exist.

Now consider applying CP together (P followed by C).

$$CP \longrightarrow \pi^+ \rightarrow \mu^+_{LH} + \nu_{\mu\ LH}$$

This works great!

Perhaps CP is what meant all along by mirror symmetry. If we do an inversion, followed by replacing each particle by its antiparticle, then we restore our *intuitive* sense. Perhaps CP is conserved in weak interactions.

CP and Kaons

Neutral kaons are pseudoscalars, so we have

$$\begin{aligned} P|K^0\rangle &= -|K^0\rangle & P|\bar{K}^0\rangle &= -|\bar{K}^0\rangle \\ C|K^0\rangle &= |\bar{K}^0\rangle & C|\bar{K}^0\rangle &= |K^0\rangle \\ CP|K^0\rangle &= -|\bar{K}^0\rangle & CP|\bar{K}^0\rangle &= -|K^0\rangle \end{aligned}$$

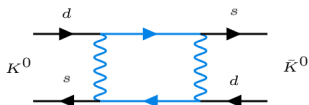
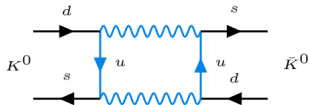
CP and Kaons

Neutral kaons are pseudoscalars, so we have

$$\begin{aligned} P|K^0\rangle &= -|K^0\rangle & P|\bar{K}^0\rangle &= -|\bar{K}^0\rangle \\ C|K^0\rangle &= |\bar{K}^0\rangle & C|\bar{K}^0\rangle &= |K^0\rangle \\ CP|K^0\rangle &= -|\bar{K}^0\rangle & CP|\bar{K}^0\rangle &= -|K^0\rangle \end{aligned}$$

These two, K^0 and $\bar{K}^0$ have no distinguishing conserved quantum number. Nothing prohibits their oscillation!

Here are two example diagrams showing how a K^0 can turn into a $\bar{K}^0$



What we observe will be a linear combination of K^0 and $\bar{K}^0$.

$$|K_1\rangle = \left(\frac{1}{\sqrt{2}}\right)(|K^0\rangle - |\bar{K}^0\rangle) \text{ and } |K_2\rangle = \left(\frac{1}{\sqrt{2}}\right)(|K^0\rangle + |\bar{K}^0\rangle)$$

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Now if CP is conserved in weak interactions, then K_1 must decay to a state with $CP = +1$ and K_2 to a state with $CP = -1$.

It turns out that the allowed decays are only

$$|K_1\rangle \rightarrow 2\pi \text{ and } |K_2\rangle \rightarrow 3\pi$$

Moreover the 2π decay is faster by a factor of 100 than the 3π decay.

CP and Kaons

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$$\text{So } |K^0\rangle = (\frac{1}{\sqrt{2}})(|K_1\rangle + |K_2\rangle) \text{ and } |\bar{K}^0\rangle = (\frac{1}{\sqrt{2}})(|K_2\rangle - |K_1\rangle)$$

If we start with a beam of $|K^0\rangle$, then we will first lose the K_1 's as they decay to 2π , and we shall be left with a beam of pure K_2 's. Near the source we shall see lots of 2π events, but farther down only the 3π events.

Thus CP invariance in weak interactions says that considering the neutral pion system, there are short-lived neutral K mesons (K_1), and long-lived neutral K mesons (K_2).

In 1956, Lederman and group found lifetimes to be

$$\tau_1 = 0.895 \times 10^{-10} \text{ s}$$

$$\tau_2 = 5.11 \times 10^{-8} \text{ s}$$

with a negligible difference in mass ($m_2 - m_1 = 3.48 \times 10^{-6} \text{ eV}/c^2$).

So is K^0 the "real" particle or K_1 ?

CP violation

CP eigenstates are $|K_1\rangle$ and $|K_2\rangle$

Since K_1 decays rapidly, let us call it K_S^0 (i.e. K^0 -short) and let us call K_2 as K_L^0 (i.e. K^0 -long).

Now, at all “late times” we expect to have only K_L^0 , and thus have only 3π decays.

But of course... experimentally we do see some 2π decays at the end of a beam (at a late time!).

Thus, $|K_L^0\rangle \neq |K_2\rangle$, but rather $|K_L^0\rangle = \frac{1}{\sqrt{1+|\epsilon|^2}}(|K_2\rangle + \epsilon|K_1\rangle)$

The long-lived neutral kaon is *not* a perfect eigenstate of CP, but contains a little bit of an admixture.

Experimentally, $\epsilon = 2.24 \times 10^{-3}$, and its non-zero value is a measure of nature's departure from perfect CP invariance.

CP violation

There is another way that K_L^0 can decay

$$K_L^0 \rightarrow \pi^+ + e^- + \bar{\nu}_e$$

$$K_L^0 \rightarrow \pi^- + e^+ + \nu_e$$

CP takes one to the other, so if *CP* is conserved, then both of these decays should be equal.

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Clear distinction between matter and antimatter. This CP violation is now observed in several other decays such as neutral B mesons (and recently in D mesons). We have several sources for CP violation in standard model, however they are insufficient to explain the observed matter-antimatter asymmetry in the universe...

This brings us to T and CPT

Time reversal symmetry : run the laws of physics backwards in time.

For the longest time, T was just taken as granted to be a symmetry.

No evidence of T violation in strong and electromagnetic interactions.

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TCP theorem is one the deepest results in QFT - impossible* to write a QFT where TCP is not conserved. TCP conservation requires every particle and antiparticle to have precisely the same mass and lifetime.

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Results from “[Observation of Time-Reversal Violation in the \$B^0\$ Meson System](#)”, The BaBar Collaboration, PRL **109**, 211801 (2012) and “[Time Reversal Violation from the entangled \$B^0\bar{B}^0\$ system](#)”, J. Bernabéu et al., JHEP **08**, 064 (2012).

Time reversal violation

Using the $\Upsilon(4S) \rightarrow B_1 B_2$ decay: this system can be written as

$$|i\rangle = \frac{1}{\sqrt{2}} (B^0(t_1)\bar{B}^0(t_2) - \bar{B}^0(t_1)B^0(t_2))$$
$$|i\rangle = \frac{1}{\sqrt{2}} (B_+(t_1)B_-(t_2) - B_-(t_1)B_+(t_2))$$

where $t_2 > t_1$

Given $B_1 B_2$, if one tags B_1 at time t_1 , then one knows what B_2 is at time t_1 by entanglement.

$B_{\pm}$ can be tagged by decay $B_{\pm} \rightarrow J/\psi + K_S^0$ ($K_S^0 \rightarrow \pi\pi$)
 B_0 can be tagged by decay $B_0 \rightarrow \ell^+ X$.

Start with $\Upsilon(4S) \rightarrow B_1 B_2$, tag $B_1(t_1)$ as $B_{\pm}$ or $B_0/\bar{B}_0$.
This tells us what $B_2(t_1)$ is.

Then watch the decay and get $B_2(t_2)$.

Time reversal violation

Reference		T -conjugate	
Transition	Final state	Transition	Final state
$\bar{B}^0 \rightarrow B_-$	$(\ell^+ X, J/\psi K_S)$	$B_- \rightarrow \bar{B}^0$	$(J/\psi K_L, \ell^- X)$
$B_+ \rightarrow B^0$	$(J/\psi K_S, \ell^+ X)$	$B^0 \rightarrow B_+$	$(\ell^- X, J/\psi K_L)$
$\bar{B}^0 \rightarrow B_+$	$(\ell^+ X, J/\psi K_L)$	$B_+ \rightarrow \bar{B}^0$	$(J/\psi K_S, \ell^- X)$
$B_- \rightarrow B^0$	$(J/\psi K_L, \ell^+ X)$	$B^0 \rightarrow B_-$	$(\ell^- X, J/\psi K_S)$

$B_2(t_1) \rightarrow B_2(t_2)$ is the rate, which we can measure to see if the rate is the same in both directions or not.

Results show a T invariance violation in the B meson system with significance equivalent to 14σ .

CP is violated, T is violated. But CPT holds as a symmetry of all interactions.