PhD Topology
Drafted on February 2025

Semester I

Smooth structures, Smooth maps, Bump functions
Partitions of Unity, Extension Lemma, Tangent Vectors, Pushforwards
Tangent Bundles, Vector fields, Vector Bundles, Bundle Maps, Sections,
Cotangent Bundles
Inverse/Implicit Function Theorem, Submanifolds, Submersions, Level
sets, Statement of Sard's Theorem
**Integral curves, flows, Lie bracket of vector fields, Statement of Frobenius Theorem*

Differential Forms, Wedge product, Exterior Derivative, Definition of
De Rham Cohomology groups
Orientation, Riemannian Volume form
Integration on Manifolds using differential forms, Stoke's Theorem

*new

Semester II

Fundamental groups and its properties, Fundamental group of the circle,
Brouwer's fixed point for 2-disks.
Van Kampen Theorem, Fundamental group of CW-complexes
Covering spaces, Lifting properties
Classification of covering spaces, deck transformations

Higher homotopy groups, Commutativity of higher homotopy groups,
Higher homotopy groups of covers

Delta Complexes, Simplicial and Singular Homology, Relation between
Homology and Fundamental group
Homotopy Invariance, Relative Homology, Exact sequences, Brouwer's
fixed point for n-disks,
Excision, Mayer-Vietoris, Statement of Equivalence of simplicial and
singular homology

CW-complexes, Degree, Cellular homology, Euler Characteristic
Homology with coefficients, Invariance of domain,
Orientation of topological manifolds, Orientation in terms of Homology

Idea of Cohomology, Statement of Poincare Duality
Universal Coefficients Theorem
Cohomology of spaces, Cross and Cup products,
Cohomology Ring, Statement of the basic Kunneth Formula,

References

A. Hatcher, "Algebraic Topology"
J. Lee, "Introduction to Smooth Manifolds"
G. Bredon, "Topology and Geometry"
Greenberg and Harper, "Algebraic Topology: A first course"
Guilleman and Pollack, "Differential Topology"

PhD Topology: Previous Syllabus

Semester I

Smooth structures, Smooth maps, Bump functions
Partitions of Unity, Extension Lemma, Tangent Vectors, Pushforwards
Tangent Bundles, Vector fields, Vector Bundles, Bundle Maps, Sections, Cotangent Bundles
Inverse/Implicit Function Theorem, Submanifolds, Submersions, Level sets, Statement of Sard's Theorem
Differential Forms, Wedge product, Exterior Derivative, Definition only of De Rham Cohomology groups
Orientation, Riemannian Volume form
Integration on Manifolds using differential forms, Stoke's Theorem
Fundamental groups and its properties, Fundamental group of the circle, Brouwer's fixed point for 2-disks.
Van Kampen Theorem, Fundamental group of CW-complexes
Covering spaces, Lifting properties
Classification of covering spaces, deck transformations
Higher homotopy groups, Commutativity of higher homotopy groups, Higher homotopy groups of covers

Semester II

Delta Complexes, Simplicial and Singular Homology, Relation between Homology and Fundamental group
Homotopy Invariance, Relative Homology, Exact sequences, Brouwer's fixed point for n-disks
Excision, Mayer-Vietoris, Statement of Equivalence of simplicial and singular homology
CW-complexes, Degree, Cellular homology, Euler Characteristic
Homology with coefficients, Axioms for Homology, Statement of Lefschetz

fixed point theorem, Statement of Invariance of domain,
Orientation of topological manifolds, Orientation in terms of Homology
Idea of Cohomology, Statement of Poincare Duality
Universal Coefficients Theorem
Cohomology of spaces, Cross and Cup products
Cohomology Ring, Statement of the basic Kunneth Formula,
Computation of Cohomology rings, Statement of DeRham comparison theorem

References

A. Hatcher, "Algebraic Topology"
J. Lee, "Introduction to Smooth Manifolds"
G. Bredon, "Topology and Geometry"
Greenberg and Harper, "Algebraic Topology: A first course"
Guilleman and Pollack, "Differential Topology"