Quiz 4

November 6, 2015

Blanket assumption: We work over the field of rationals, $\mathbb{Q}$.

The following problems is a single problem broken up into steps. So notations and hypotheses in each problem carries on to the subsequent problems.

Problem 1. Let ω be a primitive 11-th root of unity. Let $x = \omega + \frac{1}{\omega}$. For $i = 2, 3, 4, 5$ write $\alpha_i := \omega^i + \frac{1}{\omega^i}$ as an i degree polynomial in x. (For example, $\alpha_2 = x^2 - 2$. Why?) (3)

Problem 2. Noting that $\alpha_1 = x$, prove that $1 + \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 = 0$. Substituting the polynomials from the previous problem show that x satisfies the polynomial:

$$P(X) = X^5 + X^4 - 4X^3 - 3X^2 + 3X + 1.$$

(Don't worry if there is a slight mistake and you do not get the exact form above. If the idea is correct you get full score.) (3)

Assume that the P is irreducible and is the minimal polynomial of x over $\mathbb{Q}$.

Problem 3. Prove that the Galois group of $\mathbb{Q}(x)/\mathbb{Q}$ is $\frac{\mathbb{Z}}{5\mathbb{Z}}$. (4)

Problem 4. Is this Galois group solvable? (0)