Quiz 3

October 16, 2015

Problem 1. Suppose $p(X)$ be a cubic polynomial over some field F such that it has multiple roots in the splitting field. Show that $p(X)$ has a linear factor in $F[X]$. Is the same true for quartics (degree 4 polynomials)? (3)

Problem 2. Suppose x, y and z are complex numbers such that they satisfy the following three equations:

$$\begin{aligned} x + y + z &= 1 \\ x^2 + y^2 + z^2 &= 2 \\ x^3 + y^3 + z^3 &= 3 \end{aligned}$$

Take it for granted that x, y and z can be irrational. Prove that $x^n + y^n + z^n$ is rational for every n.(hint[1]) (4)

Problem 3. Prove that an extension of degree 2 is Galois. (3)

[1] *Hint:* Consider the field of symmetric functions we saw in the last tutorial.