Quiz 2
September 18, 2015

Note: You can use any result used in class as long as you clarify which result you are using. The number in brackets indicate the number of points.

Problem 1. Prove that $f(X) = X^2 - 2X - 2$ is irreducible in $\mathbb{Q}[X]$.
(2)

Problem 2. Suppose θ is a root of $f(X)$ in problem 1. Write

$$\frac{\theta}{\theta + 1} \in \mathbb{Q}(\theta)$$

as a polynomial in θ.
(4)

Problem 3. Suppose $f(X)$ be an irreducible polynomial of degree n. Let E/F be the splitting field of $f(X)$. Prove that $n | [E : F]$. If f is separable, prove that $n | \operatorname{Gal}(E/F)$.
(4)