QUIZ 1
August 21, 2015

This whole quiz revolves around the ring

$$R = \frac{\frac{\mathbb{Z}}{2\mathbb{Z}}[X]}{(X^2 + X + 1)}$$

Problem 1. Prove that R is a finite ring. List its elements.

Problem 2. Prove that R is an integral domain.

Problem 3. Prove that the non-zero elements of R have inverses, and hence it is a field.

Problem 4. Can you give an theoretical argument (which does not involve computing inverses) to say that R is a field?