Assignment 7

October 9, 2015

Problem 1. Suppose F has characteristic n where $n \neq 2$. Suppose E/F is a field extension with $[E : F] = 2$. Prove that E/F is a Galois extension.

Problem 2. Prove that the two field extensions E/F and K/E are Galois need not imply that K/F is Galois. (For example, look at $\mathbb{Q} \subset \mathbb{Q}(\sqrt{2}) \subset \mathbb{Q}(\sqrt[4]{2})$.)

Problem 3. Prove that if E/F is a Galois extension and $p(X) \in F[X]$ is irreducible, then all the irreducible factors of $p(X)$ in $E(X)$ have the same degree.

Problem 4. Check that $\mathbb{Z}$ with the partial order $m \leq n$ whenever $m \mid n$ defines a lattice. What are $a \wedge b$ and $a \vee b$ for $a, b \in \mathbb{Z}$ (in terms of elementary number theoretic functions)?