Assignment 2

August 21, 2015

Problem 1. If R is a field, prove that the map $R \to \mathrm{Frac}(R)$ given by $a \mapsto a/1$ is an isomorphism.

Problem 2. If R and S are domains and $\varphi : R \to S$ is an isomorphism. Prove that there is an isomorphism $\hat{\varphi} : \mathrm{Frac}(R) \to \mathrm{Frac}(S)$ given by $\hat{\varphi}(a/b) = \varphi(a)/\varphi(b)$.

Problem 3. If R is a ring, prove that $\varphi : R[X] \to R$ given by $\varphi(f(X)) = f_0$, where $f(X) = f_0 + f_1X + \cdots + f_nX^n$ is a ring map. What is $\ker \varphi$?

Problem 4. Prove that if R is a commutative ring with 1 and $I \subset R$ is an ideal, then there is a one to one correspondence

$$\left\{ \begin{matrix} \text{ideals } J \text{ in } R \\ \text{such that } I \subset J \subset R \end{matrix} \right\} \leftrightarrow \left\{ \text{ideals } \bar{J} \text{ of } R/I \right\}$$

given by $J \mapsto J/I$ and $\bar{J} \mapsto J = \left\{ r \in R \mid |[r] \in \bar{J} \right\}$.

Problem 5. Use Euclidean algorithm to find the gcd of $X^4 + X^2 + 1$ and $X^3 - 1$ in $\mathbb{Q}[X]$. Write the gcd as a $\mathbb{Q}[X]$ linear combination of the two given polynomials.

Problem 6. Consider $X^4 + X^2 + 1$ and $X^3 - 1$ as elements of $\frac{\mathbb{Z}}{3\mathbb{Z}}[X]$. Are these irreducible? Do these polynomials split in $\frac{\mathbb{Z}}{3\mathbb{Z}}[X]$? If so what are the linear factors?