End Sem : MTH 103 : Introduction to Computing

April 26, 2014

Please take note of the following.

1. Answer all the questions.
2. Each question is worth 20 points. The distribution of points among subparts are mentioned after each question.
3. You can use functions from python packages, but nothing more while writing algorithms.
4. For the computational problems, show the steps clearly.

1) Consider the following python function

```
def f(m, n) :
        if m >= n and n >= 0 :
                if n == m or n == 0 :
                        rval = 1
                else :
                        rval = f(m-1, n-1) + f(m-1, n)
        else :
                rval = None
        return rval
```

   a) Compute $f(4,0), f(4,1), f(4,2), f(4,3), f(4,4)$. $(1 + 4 + 8 + 4 + 1 = 18)$

   b) Can you guess what f is? (2)

2) Given a function f which takes two integers and *returns* a single integer, and a list l, define a function *fold(f, l)* which returns
   - 0 if the list is empty
   - the element of the list if the list has exactly one element
   - $f(\ldots f(f(l_1, l_2), l_3), \ldots, l_n)$ if the list has n elements

   $$l = [l_1, l_2, \ldots, l_n].$$

   For example, if *add* is a function such that `add(m, n)` returns `m + n`, `fold(add, [1,2,3,4])` should return the sum of the elements of the list, namely, 10. (20)

3) Answer the following.
   a) Find a polynomial $p(x)$ which passes through the points $(-2, 0)$, $(0, 0)$ and $(2, 4)$ using Lagrange interpolation formula. (10)

   b) Find the approximate derivate of p at -1 using the numerical difference method with $h = .1$. (5)

c) *Sweep* the first column of the following matrix as the $(1,1)$-th entry as pivot,

$$\begin{pmatrix} 2 & 8 & 6 & 0 \\ 3 & 12 & 9 & 10 \end{pmatrix}$$

Please indicate the row operations you are using. (5)

4) Consider the function $f(x) = x(x-1)(x-2)(x-3)(x-4)$. Show the first three iterations of finding the integral

$$\int_0^4 f(x)dx$$

using the trapezoidal method. Here start with $h = b - a$ and halve h each time. Does it converge in two steps? If it converges, is the answer correct? $(2 + 2 + 4 + 1 + 6 = 15)$

A polynomial $a(x) = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n$ can be represented by a list $la = [a_0, a_1, \ldots, a_n]$. Similarly any such list represents a polynomial. Given a list l and a number x, write an algorithm to compute $l(x)$, the value of the polynomial represented by l at x. (5)

5) This has two parts.

a) Write a code (function), called `clear_extra_space` which given a string as its argument, removes all extra blanks in between words. For example, `clear_extra_space(' This has lots of blanks. ')` will return `'This has lots of blanks.'`. Note that the initial and final blanks are all removed. (12)

b) Given a number return the middle number as explained below. Call this function `midno`. If the number of digits is divisible by 4, say has 4n digits, then divide the number into 4 blocks of consecutive n digits, and then return the middle two blocks. For example, for `midno(12345678)`, the code should divide it into blocks 12, 34, 56 and 78 and then return 3456. If the number of digits is not divisible by four, do the following:

- if the number has 4n+3 digits, pad one 0 on the left.
- if the number has 4n+2 digits pad one 0 on each side.
- if the number has 4n+1 digits, pad two 0s on the left, and one on the right.

Thus `midno(2463543) = midno(02463543) = 4635`.
Also, `midno(246353) = midno(02463530) = 4635` (8)