

Quiz 4

Date : November 18, 2013, Total time : 50 minutes, Total points : 20 points.

Name: _____ Reg.No.: _____

All the points and lines in this quiz are in \mathbb{RP}^2 .

1. Prove that the four lines

$$\begin{aligned} X - Y &= 0, & 2X - 2Y + Z &= 0, \\ X - Y + 2Z &= 0, & X - Y - Z &= 0 \end{aligned}$$

in \mathbb{RP}^2 are concurrent. What is the common point?

4 points.

2. Find the cross ratio of the above pencil.

4 points.

3. Consider the points $A = (0 : 0 : 1)$, $B = (0 : 1 : 1)$, $C = (0 : 2 : 1)$ and $D = (0 : 3 : 1)$ on the line $X = 0$. Suppose we have a projective transformation T such that

$$\begin{aligned} A' &:= T(A) = (0 : 0 : 1) \\ B' &:= T(B) = (2 : 0 : 1) \\ C' &:= T(C) = (4 : 0 : 1). \end{aligned}$$

- 3a. Consider the projectivity from $X = 0$ to $Y = 0$ which takes $A \mapsto A'$, $B \mapsto B'$ and $C \mapsto C'$. Is this projectivity a perspectivity? (If you are using a theorem, mention it clearly.)

2 points.

- 3b. Find $D' = T(D)$, providing necessary justification and without using problem 4.

6 points.

4. Suppose in the situation of the previous problem, we are also given that $T(0 : 1 : 0) = (1 : 0 : 0)$, $T(1 : 0 : 0) = (0 : 1 : 0)$ and $T(1 : 1 : 1) = (2 : 1 : 1)$. What is the matrix for T ?

4 points.