Tutorial

Solution to a question asked in the tutorial. This particular problem uses something which was not done in class and hence won't be a part of the mid-sem.

1. The question

Problem : If $ABCD$ is a Saccheri quadrilateral, show that $CD > AB$ if and only if the angles at C, D are acute.

Solution is very easy once we know which theorem to use. The theorem we use is the following.

1.0.1. **Proposition.** *Let $ABCD$ be a quadrilateral with right angles at A and B, and unequal sides AC and BD. Then the angle C is greater than the angle at D if and only if $AC < BD$.*

I'll give a proof of this, just because it is easy.

Proof. Consider the diagram 1. Suppose that $AC < BD$. Choose E on BD such that $AC = BE$.

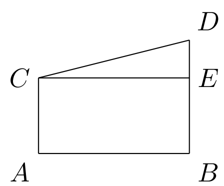

Figure 1. Proposition used in the solution {fig:propsoln}

Then $ABCE$ is a Saccheri quadrilateral. Therefore $\angle ACE = \angle CEB$. On the other hand, since $B * E * C$, $\angle ACD > \angle ACE$. Since $\angle CEB$ is an exterior angle to the triangle $\triangle CED$, $\angle CEB > \angle CDB$. Therefore, $\angle ACD > \angle ACE = \angle CEB > \angle CDB$.

Thus $AC < BD \implies C > D$.

Similarly $AC > BD \implies C < D$.

$AC = BD \implies C = D$ (being a Saccheri quadilateral).

This proves that the statement is if and only if. □

Now the proof of the problem is very easy.

Solution of the problem. Let $ABCD$ be a Saccheri quadrilateral, and suppose E and F are the mid points of AB and CD respectively. Consider figure 2. Now by a propositon we proved in class, $\angle DFE = \angle BEF = 90°$.

Now if $CD > AB$, $FD = \frac{1}{2}CD > \frac{1}{2}AB = EB$. Now $FEDB$ is a quadrilateral with F and E being one right angles. Therefore by the proposition proved above, $\angle B > \angle D$. Now the problem follows from the fact that $\angle B$ is given to be 1 right angle. □

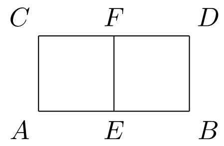

FIGURE 2. Problem of hyperbolic Sachheri quadrilaterals {fig:saccprob}