Assignment 3
Due date : November 4, 2013

Total points = 10
Penalty for submitting on November 5 : -1
Penalty for submitting on November 7 : -2
Assignments won't be accepted after November 7.

In the following problems let $M : \mathbb{R}^3 \to \mathbb{R}^3$ be an orthogonal isometry (that is, an isometry which takes 0 to 0.

1. Prove that if $(y_1, y_2, y_3) = M(x_1, x_2, x_3)$ and $(y'_1, y'_2, y'_3) = M(x'_1, x'_2, x'_3)$, then $x_1x'_1 + x_2x'_2 + x_3x'_3 = y_1y'_1 + y_2y'_2 + y_3y'_3$

2. Prove that if we write M as a matrix then $MM^t = I$, where $(\ )^t$ denote the transpose of a matrix and I is the 3×3 identity matrix.

3. Prove that $\det M = \pm 1$.

4. Suppose $\det M = 1$. Prove that 1 is an eigenvalue for M. Can you find a one dimensional subspace fixed by M. Prove that M is a rotation.

5. Write down a formula for reflection of a point along a plane given by

$$\left\{ X \in \mathbb{R}^3 \ \mid \ X \cdot A = c \right\}$$

for some fixed vector A and a fixed real number c.

6. If M is an orthogonal transformation with $\det(M) = -1$, what can you say about $M \circ R$ where R is a reflection. You can choose your reflection suitably to make computations simpler.