## Assignment 2

Due date : September 16, 2013
Total points $=10$
Penalty for submitting on September 17:-1
Penalty for submitting on September 19:-2
Assignments won't be accepted after September 19.

1. Let $\Gamma_{(a, b)}^{r}$ denote the circle with centre $(a, b)$ and radius $r . S_{(a, b)}^{r}$ be the circular inversion along $\Gamma_{(a, b)}^{r}$. Also we fix the symbols for the following operations:

$$
\begin{array}{l|l}
\hline T_{(a, b)} & \begin{array}{l}
\text { Translation by the vector }(a, b), \\
\Sigma_{r}
\end{array} \\
\text { Scaling by } r ;(x, y) \mapsto(r x, r y) .
\end{array}
$$

Prove that $S_{(a, b)}^{r}=T_{(a, b)} \circ \Sigma_{r} \circ S_{(0,0)}^{1} \circ \Sigma_{1 / r} \circ T_{(-a,-b)}$.
2. Using the above exercise, write a formula for $S_{C}^{r}$ where $C$ is the complex number $C=a+i b$ as

$$
S_{C}^{r}(Z)=\text { some complex expression in } C, Z \text { and } r .
$$

3. Complete the proof that circular inversion is conformal.
4. When is $S_{C}^{r}$ of $\Gamma_{(0,0)}^{1}$ a straight line? When is it a circle? In the cases it is a circle, what is its centre and radius?
