

### Universal Coefficient theorem continued

This lecture is a tutorial on the proof of the following version of Universal Coefficient Theorem.

**Theorem 1** (UCT-II). *Let  $C_*$  be a complex of free abelian groups. Let  $G$  be any abelian group. Then we have a short exact sequence*

$$0 \rightarrow H_n(C_*) \otimes G \rightarrow H_n(C_* \otimes G) \rightarrow \text{Tor}_1(H_{n-1}(C_*), G) \rightarrow 0$$

*which is functorial in  $C_*$  and  $G$ . Moreover this sequence splits functorially in  $G$  but the splitting is not functorial in  $C_*$ .*

Recall

**Definition 2.** *Singular homology with coefficients in  $G$  is defined as*

$$H_n(X, A; G) = H_n(\Delta_*(X, A) \otimes G).$$

**Corollary 3.** *Take  $C_* = \Delta_*(X, A)$ . Then the sequence becomes*

$$0 \rightarrow H_n(X, A) \otimes G \rightarrow H_n(X, A; G) \rightarrow \text{Tor}_1(H_{n-1}(X, A), G) \rightarrow 0$$

*which is functorial in  $(X, A)$  and  $G$ . Further it splits, where the splitting is functorial in  $G$  but not in  $(X, A)$ .*