Quiz 2

February 20, 2013

Answer all the questions. Try to be as precise as possible.
Write your name on your answer script.
Time: 30 minutes.

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1. Take a deep breath, exhale slowly. (*0 point.*)

2. In this exercise we shall compute $\mathrm{Ext}^i_{\mathfrak{AbelianGroups}}(\mathbb{Z}/p\mathbb{Z},\, \_)$ which we shall denote by $\mathrm{Ext}^i(\mathbb{Z}/p\mathbb{Z},\, \_)$ for simplicity.

(i) Write a projective resolution of $\mathbb{Z}/p\mathbb{Z}$ consisting only of $\mathbb{Z}$'s. (*1 point.*)

(ii) Compute $\mathrm{Ext}^i(\mathbb{Z}/p\mathbb{Z},\, B)$ for an abelian group B for $i \geq 0$. (*4 points.*)

3. Let R be a commutative ring with 1. Let A, B, M, P_0 and P_1 be R-modules.

Suppose $0 \to M \to P_1 \to P_0 \to A \to 0$ is an exact sequence with P_0 and P_1 projective. Prove that

$$\mathrm{Tor}^R_i(B,\, M) \simeq \mathrm{Tor}^R_{i-2}(B,\, A) \ \forall\ i \geq 3,$$

and

$$\mathrm{Tor}^R_2(B,\, A) \simeq \ker\big(B \otimes M \to B \otimes P_1\big).$$

You can ask for a hint, but that will cost you 1 point. (*5 points.*)

Hint for problem 2. Let $K = \ker(P_0 \to A)$. Break up the given exact sequence into two short exact sequences

$$0 \to M \to P_1 \to K \to 0$$
$$0 \to K \to P_0 \to A \to 0.$$

Now try to argue using what you know.