Assignment 2

April 12, 2013

Due date : April 19, 2013

Answer all the questions. Try to be as precise as possible. And please write your own answers, and don't blindly copy.

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1. Prove that $\mathbb{R}^k$ and $\mathbb{R}^l$ are not homeomorphic for $k \neq l$.

2. Solve the following problems in order[1].

(i) Suppose $X = \partial W$ where W is a compact manifold. Let $f : X \to Y$ be a smooth map. If ω is a closed k form on Y with $k = \dim X$, and if f extends to the whole of W, then

$$\int_X f^*\omega = 0.$$

(ii) Suppose that $f_0, f_1 : X \to Y$ are homotopic maps. Also suppose that X is a compact manifold without boundary (closed). Let $k = \dim X$. Prove that for all *closed* k forms ω on Y

$$\int_X f_0^*\omega = \int_X f_1^*\omega.$$

(iii) Show that if X is a simply connected manifold, then for all closed 1-forms ω and for all closed curves $\gamma \subset X$,

$$\int_\gamma \omega = 0.$$

3. Show that if $\widetilde{H}^n(X;\mathbb{Q})$ and $\widetilde{H}^n(X;\mathbb{Z}/p\mathbb{Z})$ are *zero* for all n and for all primes p, then $\widetilde{H}_n(X;Z) = 0$ for all n and hence $\widetilde{H}^n(X;G) = 0$ for all G and all n.

4. Compute all the homologies and cohomologies of $S^n \times S^m$ with $\mathbb{Z}/p\mathbb{Z}$ coefficients where p is a prime.

5. Can you find the cup product in the above example when $m = 2n+1$, $n \geq 2$? What happens when $m = 2n$?

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[1]Well, it is equally fine if you do it correctly in some other order.