

## Assignment 1

April 4, 2013

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**Due date :** April 12, 2013

Answer all the questions. Try to be as precise as possible. And please write your own answers, and don't blindly copy. ☺

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1. Write down a CW structure for the torus. (2 points.)
- 2 (Hatcher, problem 5, page 205). Regarding a cochain  $\varphi \in \Delta^1(X; G)$  as a function from paths in  $X$  to  $G$ , show that if  $\varphi$  is a cocycle, then
  - (i)  $\varphi(f \cdot g) = \varphi(f) + \varphi(g)$ ;
  - (ii)  $\varphi$  is 0 on constant paths;
  - (iii)  $\varphi(f) = \varphi(g)$  if  $f \simeq g$ .
  - (iv)  $\varphi$  is a coboundary if and only if  $\varphi(f)$  depends only on the endpoints of  $f$  for all  $f$ .(10 points.)
3. Show that, if  $f : S^n \rightarrow S^n$  is a map of degree  $d$ , then
$$f^* : H^n(S^n; G) \rightarrow H^n(S^n; G)$$
is multiplication by  $d$ . (3 points.)
4. Compute the homology groups of  $\mathbb{P}^1 \times \mathbb{P}^1$ . (5 points.)