QUANTUM FIELD THEORY
PHY 655/461

ASSIGNMENT III

(1) Express the Hamiltonian density for the free, real scalar field theory, in terms of creation and annihilation operators. Is this a finite quantity ?

Introduce the notion of *normal ordering* (denoted by : Operator products : when acting on some operator products), where the positive frequency parts (these are the components with factors like $\exp[-i\omega t]$) are sorted to the right of negative frequency parts. Show that using normal ordering on the field operator products, that define the Hamiltonian density, one may get a finite expression.

(2) Show the Lorentz-invariant nature of the measure

$$\frac{d^3p}{(2\pi)^3 2E_p}$$

(3) Derive expressions for the *equal time commutation relations* – $[\phi(\vec{x}), \phi(\vec{y})]$ and $[\phi(\vec{x}), \pi(\vec{y})]$.

(4) Derive

$$i \int d^4x e^{ipx} (\Box + m^2)\phi(x) = \sqrt{2\omega_p}\,[\hat{a}_p(\infty) - \hat{a}_p(-\infty)]$$

$$-i \int d^4x e^{-ipx} (\Box + m^2)\phi(x) = \sqrt{2\omega_p}\left[\hat{a}_p^\dagger(\infty) - \hat{a}_p^\dagger(-\infty)\right]$$

(5) Prove that

$$\lim_{\epsilon\to 0} -\frac{2\omega_k}{2\pi i}\int_{-\infty}^{\infty} \frac{d\omega}{\omega^2 - \omega_k^2 + i\epsilon} e^{i\omega\tau} = e^{-i\omega_k\tau}\Theta(\tau) + e^{i\omega_k\tau}\Theta(-\tau)$$

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* *Extra :* We introduced the Fock space and related algebra only cursorily, at a level sufficient for our purposes. If you are interested in understanding aspects of the formalism more, I encourage you to read Ch. 2 and attempt the problem on the Stone- von Neumann theorem therein, from *Condensed Matter Field Theory* by Altland and Simons.