QUANTUM FIELD THEORY
PHY 655/461

ASSIGNMENT I

(1) Derive a prediction for the blackbody spectrum, from both the classical theory and the quantum theory.

(2) Work out relations for the Einstein coefficients, without assuming an equilibrium result from the blackbody spectrum. Are ideas of second quantisation already present here ?

(3) Show that

$$\begin{aligned} \hat{a}|n\rangle &= \sqrt{n}|n-1\rangle \\ \hat{a}^\dagger|n\rangle &= \sqrt{n+1}|n+1\rangle \end{aligned}$$

(4) Show that the norm of the vector x_i is invariant under rotations. What is meant by $SO(n)$ transformations ?

(5) How is *rapidity* (β_i) defined ? Represent Lorentz transformations in terms of the rapidity.

(6) $\phi(x)$ is a scalar field. What does a change under Lorentz transformation $\phi(x^\mu) \rightarrow \phi((\Lambda^{-1})^\mu_\nu x^\nu)$ mean ? Does this mean that the scalar field changes under a Lorentz transformation ?

(7) How does a Lorentz 4-vector transform ? For two 4-vectors A_μ and B_μ, show how $A^\mu B_\mu$ transforms.

(8) Starting with the Hamiltonian for the simple harmonic oscillator and definitions for the ladder operators, show that

$$i\frac{d\hat{a}}{dt} = \omega\hat{a}$$

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(9) Derive an expression for $\langle\vec{p}\,|\vec{k}\rangle$. Use this result to motivate the identity operator for the *one-particle states*

$$\mathbb{I} = \int \frac{d^3q}{(2\pi)^3}\frac{1}{2\omega_q}|\vec{q}\rangle\langle\vec{q}|$$

(10) What is the mode expansion for a quantum scalar field $\hat{\phi}_0(x)$? What is the state $\hat{\phi}_0(x)|0\rangle$. Starting with a simple harmonic oscillator inspired Hamiltonian for the free theory (H_0), compute $[H_0, \hat{\phi}_0(x)]$.

(11) Derive expressions for the *equal time commutation relations* – $[\phi(\vec{x}), \phi(\vec{y})]$ and $[\phi(\vec{x}), \pi(\vec{y})]$.