QUANTUM FIELD THEORY
PHY 655/461

ASSIGNMENT I

(1) Utilising ordinary index notations, for instance, to express scalar and cross-products as– $\vec{A}\cdot\vec{B} = A_i B_i$; $(\vec{A}\times\vec{B})_k = \epsilon_{ijk} A_i B_j$ – find and interpret the expressions for
(a) $\vec{A}\times(\vec{B}\times\vec{C})$
(b) $\vec{A}\cdot(\vec{B}\times\vec{C})$
(c) Det$\bar{\bar{M}}$, where $\bar{\bar{M}}$ is a square matrix.

(2) Show that the norm of the vector x_i is invariant under ordinary rotations. What is meant by $SO(N)$ transformations ?

(3) Show that the Laplacian $\vec{\nabla}^2$ is invariant under ordinary rotations.

(4) How is *rapidity* $(\vec{\beta})$ defined ? Represent Lorentz transformations in terms of rapidity.

(5) How does a 4-vector field and scalar field transform under Lorentz transformations ? For two 4-vectors A_μ and B_μ, show how $A^\mu B_\mu$ transforms.

(6) How is $g_{\mu\nu}$ and $g^{\mu\nu}$ related to each other ?

(7) For the Lorentz transformation matrix $\bar{\bar{\Lambda}}$ and Minkowskian metric tensor matrix $\bar{\bar{g}}$, show that

$$\bar{\bar{\Lambda}}^T \ \bar{\bar{g}} \ \bar{\bar{\Lambda}} = \bar{\bar{g}}$$

(8) Show that

$$\left(\Lambda^{-1}\right)^{\alpha}{}_{\beta} = \left(\Lambda\right)_{\beta}{}^{\alpha}$$

(9) Show that $\Box$ is a Lorentz invariant operator.

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(10) The Lagrangian density for complex scalar fields

$$\mathcal{L} = |\partial_\mu \phi|^2 - m^2 \phi^2$$

is invariant under some internal field symmetry transformation. Identify this internal symmetry and derive the conserved Noether charge. What are the Euler-Lagrange equations of motion ?