QUANTUM FIELD THEORY
PHY 655/461

ASSIGNMENT VII

(1) Solve the Dirac equation in the rest frame and in the boosted frame.

(2) Show that

$$\begin{aligned}\bar{u}_s(p)u_{s'}(p) &= 2m\delta_{ss'} \\ \bar{v}_s(p)v_{s'}(p) &= -2m\delta_{ss'}\end{aligned}$$

$$\begin{aligned}u_s^\dagger(p)u_{s'}(p) &= 2E_p\delta_{ss'} \\ v_s^\dagger(p)v_{s'}(p) &= 2E_p\delta_{ss'}\end{aligned}$$

(3) Show that the spinor outer products satisfy

$$\begin{aligned}\sum_{s=1}^{2} u_s(p)\bar{u}_s(p) &= \not{p} + m \\ \sum_{s=1}^{2} v_s(p)\bar{v}_s(p) &= \not{p} - m\end{aligned}$$

(4) Starting from the general solution of the Dirac equation, show that

$$\begin{aligned}u_s(p) &= -i\gamma^2(v_s(p))^* \\ v_s(p) &= -i\gamma^2(u_s(p))^*\end{aligned}$$

[Hint: $\xi^{-s} = -i\sigma_2(\xi^s)^*$]

(5) What are the fourier mode expansions for ψ and $\bar{\psi}$? Postulating the equal-time anticommutation relations

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\begin{aligned}
\{\psi_a(\vec{x}), \psi_b^\dagger(\vec{y})\} &= \delta_{ab}\, \delta^{(3)}(\vec{x}-\vec{y}) \\
\{\psi_a(\vec{x}), \psi_b^(\vec{y})\} &= \{\psi_a^\dagger(\vec{x}), \psi_b^\dagger(\vec{y})\} = 0
\end{aligned}
$$

derive the anticommutation relations satisfied by the Fock space operators.

(6) What is the conjugate momenta corresponding to ψ ? What is the Hamiltonian. Express the Hamiltonian in terms of the Fock space operators.

(7) Derive an expression for $\langle 0|T\{\psi(x)\bar{\psi}(y)\}|0\rangle$. What is this object ? What happens if you choose the wrong commutation relation ?