QUANTUM FIELD THEORY
PHY 655/461

ASSIGNMENT VI

(1) Show that in the 4-vector representation, the infinitesimal Lorentz transformations may be written as

$$\begin{aligned}\delta X_0 &= \beta_i X_i \\ \delta X_i &= \beta_i X_o - \epsilon_{ijk}\theta_j X_k\end{aligned}$$

(2) Derive the Lorentz group generators J_i and K_i in the 4-vector basis. Is this representation unitary ?

(3) Why is $so(1,3) = su(2) \oplus su(2)$? Work out the decomposition of the irreducible representations of the Lorentz algebra into the irreducible representations of its $so(3)$ subalgebra.

(4) What are the spinor representations corresponding to $(0, \frac{1}{2})$ and $(\frac{1}{2}, 0)$?

(5) Calculate explicitly, for Lorentz transformations, the quantities $\delta(\psi_L^\dagger \psi_L)$, $\delta(\psi_R^\dagger \psi_L)$ and $\delta(\psi_L^\dagger \sigma_k \psi_L)$. Motivate a Lagrangian for spin-1/2 particles based on these Lorentz transformations.

(6) Derive the equations of motion for the ψ and $\bar{\psi}$ fields from the Dirac Lagrangian

$$\mathcal{L} = \bar{\psi}\,(i\gamma^\mu \partial_\mu - m)\,\psi$$

(7) Derive the Lorentz generators $S_{\mu\nu}$ in the Weyl representation of the gamma matrices.

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(8) Show the following

$$\begin{aligned} \{\gamma^\mu, \gamma^\nu\} &= 2g^{\mu\nu} \\ (\gamma^0)^\dagger &= \gamma^0 \; ; \; (\gamma^k)^\dagger = -\gamma^k \\ \gamma^0 (\gamma^\mu)^\dagger \gamma^0 &= \gamma^\mu \end{aligned}$$

(9) How is minimal coupling of photons to fermions incorporated ? Show that

$$\not{D}^2 = \mathcal{D}^2 + \frac{e}{2} F_{\mu\nu} \sigma^{\mu\nu}$$

What is the new equation of motion ?

Extra : Attempt problem (10.1) from the textbook *Quantum Field Theory and the Standard Model, Matthew D. Schwartz.*