QUANTUM FIELD THEORY
PHY 655/461

ASSIGNMENT II

(1) The *Higgs sector* of the Standard Model of particle physics gives elementary particles their masses. The Lagrangian of this sector has a form very similar to

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - \frac{\mu^2}{2}\phi^2 - \frac{\lambda}{4!}\phi^4$$

(a) What is the conjugate momentum operator corresponding to ϕ ? What are the canonical commutation relations ?
(b) Corresponding to this Lagrangian density, what is the action, Hamiltonian density and Hamiltonian ?
(c) Use Euler-Lagrange equations to derive the equation of motion.

(2) The Lagrangian

$$\mathcal{L} = |\partial_\mu\phi|^2 - m^2\phi^2$$

is invariant under an internal symmetry transformation $\phi \to e^{-i\theta}\phi\ \forall\theta \in \mathbb{R}$. Derive the conserved current. What is the expression for the Noether charge ?

(3) Let $\Theta_{\mu\nu}$ be the canonical energy-momentum tensor. Consider a modified tensor

$$\mathcal{T}_{\mu\nu} = \Theta_{\mu\nu} + \partial^\rho\chi_{\rho\mu\nu}$$

(a) If we choose a tensor such that $\chi_{\lambda\alpha\beta} = -\chi_{\alpha\lambda\beta}$, is the conservation law modified ?
(b) What were the conserved Noether charges for $\Theta_{\mu\nu}$. Do the conserved charges change under this modification ?

This additional freedom may be used to construct a modified symmetric energy-momentum tensor such that $\mathcal{T}_{\mu\nu} = \mathcal{T}_{\nu\mu}$ [J. Belinfante, Physica **6**, 887 (1939)]

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