MTH 620: Quiz -1

(1) (10 marks) Prove that a sequence $M' \xrightarrow{f} M \xrightarrow{g} M'' \to 0$ is exact $\iff$ for all modules N, the sequence

$$0 \to \mathrm{Hom}(M'', N) \xrightarrow{g^*} \mathrm{Hom}(M, N) \xrightarrow{f^*} \mathrm{Hom}(M', N)$$

is exact.

(2) (10 marks) Show that if $f : A \to B$ is a ring homomorphism and $\mathfrak{p} \subset A$ is a prime ideal then $\mathfrak{p}$ is the contraction of a prime ideal in $B \iff \mathfrak{p} = \mathfrak{p}^{ec}$.