PHY 455 More questions (relevant for exam):

1. Consider the Schwarzschild spacetime, and a particle of Newtonian mass $m = 1$ in this spacetime. The particle is in *radial free fall* (i.e., only radial coordinate changes along the trajectory) starting from rest at infinity. Starting from rest at infinity implies that as $r \to \infty$, $dt/d\tau \to 1$ (due to the fact that the metric approaches Minkowski in this limit, and for a particle at rest in Minkowski spacetime, proper time τ equals coordinate time t). Use the equations we derived for particle orbits in the Schwarzschild spacetime, and find out the values of the conserved quantities L and E along this orbit.

2. In the previous example of radial free fall, find the four-velocity $u^\alpha = \frac{dx^\alpha}{d\tau}$ at any arbitrary point along the particle orbit. The correct answer should be $u^\alpha = ((1 - 2GM/r)^{-1}, -\sqrt{2GM/r}, 0, 0)$.

3. For the particle in radial free fall as above, how much time does it take in the particle's clock, to pass between the radii $6GM$ and $2GM$?

4. For the Robertson-Walker universe with $\kappa = 0$, the metric is

$$ds^2 = -dt^2 + a^2(t)(dr^2 + r^2(d\theta^2 + d\phi^2)). \tag{1}$$

   Compute the Ricci tensor (all components).

5. The Friedmann equations for $a(t)$ with general κ are (notation: $\dot{a} = \frac{da}{dt}$):

$$\begin{aligned} (\frac{\dot{a}}{a})^2 &= \frac{8\pi G\rho}{3} - \frac{\kappa}{a^2}; \\ \frac{\ddot{a}}{a} &= -\frac{4\pi G(\rho + 3p)}{3}. \end{aligned} \tag{2}$$

   Now consider $\kappa = 0$. Let the universe be radiation dominated - so $p = \frac{\rho}{3}$, and $\rho = \frac{C}{a^4}$ where C is a constant. Solve for $a(t)$ and show there is a Big bang at $t = 0$. Does the universe expand forever according to this metric?