PHY 455 Assignment 2

Marking: Total marks : 50.

Consider the following metric in 3-dimensional spacetime (i.e., one dimension lower than our spacetime):

$$ds^{2} = -(r^{2} - M)dt^{2} + \frac{dr^{2}}{(r^{2} - M)} + r^{2}d\phi^{2};$$

where M > 0, $r > \sqrt{M}$ and $\phi \in [0, 2\pi]$.

(a) Find the Christoffel symbols, Ricci tensor and Ricci scalar for this metric. (20 marks)

(b) Does this metric solve Einstein equation in vacuum? Or with matter? Or with cosmological constant? Support your answer by computing the Einstein tensor for this metric. (10 marks)

(c) Find conserved quantities along the path of a free massive particle in this spacetime. Using this, derive an equation which should be followed by the free massive particle in terms of an effective potential in this spacetime exactly as we have done for the Schwarzschild spacetime. (20 marks)