PHY 455 Assignment 1 (special relativity) :

Marking: Total marks : 50 - Questions 1 and 2 worth 10 marks each, Question 3 worth 6 marks and Question 4 worth 24 marks (each subquestion worth 4 marks).

1. There is an interplanetary rocket race between *Team England* and *Team India* both of which we can assume to be travelling in the $+x$ direction. *Team England* is travelling in an old rocket at speed $0.9c$ relative to the stationary referee in the finish line. *Team England* see faster *Team India* passing past them at speed $0.9c$ (this speed is as observed by *Team England*). What is the speed of *Team India* relative to the referee in the finish line? Remember, nobody can travel faster than speed c, not even *Team India*!

2. Consider two events A at (x, ct) and B at $(x+dx, c(t+dt))$ in spacetime. Assume that both events happen at the same y and z coordinates, so that we can ignore them for simplicity. With respect to the inertial observer at the origin of the coordinate system, you are given that event A happens before event B. You are also given that A and B are timelike-separated ($ds^2 < 0$ between them). Show that with respect to any other inertial observer travelling with speed less than c in either the $+ x$ or $-x$ direction, A still happens before B. However, if we allow an inertial observer moving with speed greater than c (i.e., a coordinate frame attached to a particle that always moves at speed greater than c), then show that from the point of view of this observer, B can happen before A.

3. Consider a Lorentz transformation (boost) in the x-ct plane. Draw the x' and ct' coordinate axes in the x-ct plane.

4. A particle always moving faster than light is called a tachyon. Let ds^2 denote the line element/metric. Through the exercises below (and from the answer to question 2), explore why tachyons are nasty, undesirable creatures which shouldn't exist in a reasonable universe:

a) Show that the worldline of this particle is spacelike.
b) By analogy with timelike worldlines, we can parametrize this worldline by s, the spacelike interval along the trajectory. Define the four-velocity $\bar{\mathbf{u}}$ by $u^\alpha = \frac{dx^\alpha}{ds}$. Show that $\bar{\mathbf{u}} \cdot \bar{\mathbf{u}} = 1$.
c) Evaluate the components of the tachyon's four-velocity $\bar{\mathbf{u}}$ in terms of its three-velocity $\vec{V} = \frac{d\vec{x}}{dt}$.
d)Define four-momentum of the tachyon by $\bar{\mathbf{p}} = m\bar{\mathbf{u}}$ and by analogy with particles moving on timelike worldlines, define its energy and relativistic three-momentum. Find the relation between the two.
e) Show that there is an inertial frame where the energy of a tachyon is negative.
f) Argue from e) that if tachyons interact with normal particles, a normal particle could emit a tachyon with total energy and three-momentum being conserved.