Assignment 3

1. Solve the following IBVP:

$$\frac{\partial u(x,t)}{\partial t} = \frac{\partial^2 u}{\partial x^2};$$
$$0 \leq x \leq 2$$
$$u(x, t = 0) = x$$
$$u(x = 0, t) = u(x = 2, t) = 0; t > 0.$$

You may use the form of the general solution for these boundary conditions that was derived in class.

2. Do separation of variables and write the general solution to the Laplace equation in two dimensions,

$$\nabla^2 u(x, y) = 0$$

where ∇^2 is the Laplace operator in two dimensions and $0 \leq x \leq a$, $0 \leq y \leq b$. You are given the following boundary conditions for the problem:

$$u(x, y = b) = 0, \quad 0 \leq x \leq a;$$
$$\frac{\partial u}{\partial x}(x = 0, y) = 0; \frac{\partial u}{\partial x}(x = a, y) = 0. \quad 0 \leq y \leq b$$

Find the general solution obeying these boundary conditions.

3. Consider a string tied at two ends $x = 0$ and $x = \pi$. Its transverse oscillations are governed by the wave equation

$$\frac{\partial^2 u(x,t)}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2};$$

. Solve the following IBVP:

$$u(x, t = 0) = \sin(2x); \frac{\partial u}{\partial t}(x, t = 0) = 0.$$

4. Try the exercises in Chapter 4 relevant to the theorems done in class.