Assignment: Legendre polynomials

Legendre polynomials $P_n(x)$ are defined as polynomial solutions of the Legendre differential equation

$$(1-x^2)y''(x) - 2xy'(x) + n(n+1)y = 0$$

satisfying $P_n(1) = 1$. Here n is taken to be a non-negative integer.

1. Show that
$$P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n.$$
This is called *Rodrigues formula*. To do this, you have to first show that the given expression for $P_n(x)$ is really a solution to the Legendre differential equation. Then show that the given expression indeed given $P_n(1) = 1$.

2. The Associated Legendre equation is given by
$$(1-x^2)y''(x) - 2xy'(x) + [l(l+1) - \frac{m^2}{1-x^2}]y = 0.$$
Find the series solution to this DE around $x = 0$. What is the radius of convergence of this series solution in general?

3. Using $x = \cos\theta$, write the above equation as a DE in θ. Does this equation look familiar from physics?