MATH 521 : Differential Manifolds

Core course: Yes Term: Fall MWF 10:00 - 10:50 AM Instructor: Suneeta Vardarajan Office: CAB 699 e-mail: vardarajan@math.ualberta.ca Phone: 492-8455

Course description:

In this course, we will first define an abstract object — the *n*-dimensional differential (or smooth) manifold — which, roughly speaking, looks locally like a piece of \mathbb{R}^n . A manifold generalizes the notion of curves and surfaces to higher dimensions and provides a description of these geometric objects without thinking of them as embedded in a higher dimensional Euclidean space. Manifolds arise naturally in various areas of mathematics, and are also important in theories of physics such as relativity and string theory. The recent proof of the Poincáre conjecture extensively used analysis on these differential manifolds. The main goal of this course is to study calculus on manifolds, which is fundamental to all of these diverse applications. Some of the topics covered (after a detailed introduction to manifolds) are :

(i) Vector fields and their associated flows, vector bundles, Lie derivatives, Frobenius theorem.

(ii) Tensor fields and differential forms, differential and integral calculus on manifolds, Stokes theorem, Riemannian metric.

(iii) Submersions, immersions, embeddings, submanifolds, quotient manifolds.

(iv) Lie groups; actions of Lie groups on manifolds, Lie algebras.

Prerequisite: MATH 446 OR MATH 448, or the permission of the instructor (please see my office number and email at top of page to contact me).

Textbooks:

Required text — John M Lee, *Introduction to Smooth Manifolds* (available online at the U of A as an e-book but it is better to buy a copy - the online version can only be viewed page by page);

Suggested references — D Barden and C Thomas, An Introduction to Differential Manifolds; R Abraham, JE Marsden, T Ratiu, Manifolds, Tensor Analysis and Applications (this is a very comprehensive reference).

Grading: Homework 30 %, Mid-term 20 %, Final 50 %. Exam dates: TBA