# Vibrational Spectroscopy via Computer Simulations: Raman, Infra-Red and Related Methods

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MASTANI Summer School, July 11th 2014

#### Vibrational spectroscopies

- Introduction
- Infra-red spectroscopy
- Raman spectroscopy
- Using Quantum ESPRESSC

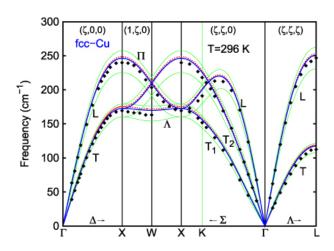
#### Introduction

- Characterisation of the system via its vibrations
- Different vibrational spectroscopies
  - Neutron scattering spectroscopy
  - Infra-red spectroscopy
  - Raman spectroscopy

# Neutron scattering

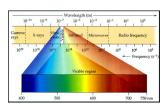
- Strong scattering at low-Z elements
- No charge
- Nuclear interaction simple
- Magnetic dipole moment, studies of magnetic structures
- Low energy, non-distructive
- Expensive to produce
- Interact weakly with matter, large samples needed

#### Neutron scattering spectroscopy: Example

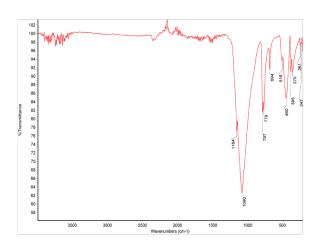


### Infra-red spectroscopy

- Dispersion of light at frequencies comparable with those of vibrational modes
- Wave lengths are macroscopic compared to the size of the interatomic distance or units cells ⇒ only q = 0
- Linear spectroscopy here; non-linear methods like multi-dimensional (several photons) like 2D-IR beyond the scope



# Infra-red spectroscopy: Example



#### Introduction

Infra-red spectroscopy Raman spectroscopy Using Quantum ESPRESSO

#### Raman effect

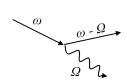


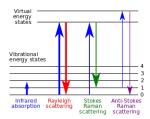


#### Raman scattering spectroscopy

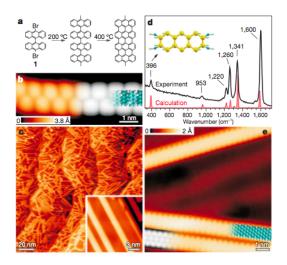
- Based on inelastic scattering of light;  $\mathbf{q} \approx 0$
- Only non-resonant processes possible nowadays
- Central quantity: polarisability tensor  $\overline{\overline{\alpha}}$
- Cross section of mode  $\nu$ :

$$\frac{d\sigma^{\nu}}{d\Omega} = \frac{{\omega_{\nu}}^{4}}{(4\pi\varepsilon_{0})^{2}c^{4}} \left| \mathbf{e}_{i} \cdot \overline{\overline{A}}^{\nu} \cdot \mathbf{e}_{s} \right|^{2}$$





#### Raman spectroscopy: Example



#### Static approach

- Harmonic theory
- Eigenfrequencies and eigenmodes calculated using DFPT/linear response, intensity needed

#### Dynamic approach

 Molecular dynamics/time evolution of dipole M (IR) or polarisability α (Raman):

$$I_{\rm IR}(\omega) \propto FT \left\{ \int_{t=0}^{\infty} M(t) M(t=0) dt \right\}$$
 $I_{\rm Raman}(\omega) \propto FT \left\{ \int_{t=0}^{\infty} \alpha(t) \alpha(t=0) dt \right\}$ 

VOLUME 88, NUMBER 17

PHYSICAL REVIEW LETTERS

29 APRIL 2002

#### Anharmonic Raman Spectra in High-Pressure Ice from Ab Initio Simulations

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#### Michele Parrinello

CSCS Swiss Center for Scientific Computing, Via Cantonale, CH-6928 Manno, Switzerland and Physical Chemistry ETH Hönggerberg HCI, CH-8093 Zurich, Switzerland (Received 30 November 2001; published 16 April 2002)

We calculate from ab initio molecular dynamics the Raman scattering of high-pressure ice. To this effect we apply a new method based on the Berry phase theory of polarization. Our results are in agreement with recent and difficult experiments and are compatible with a picture in which ice VII is a protondisordered system and in ice X the hydrogen bond is symmetric.

DOI: 10.1103/PhysRevLett.88.176401

PACS numbers: 71.15.Pd, 62.50.+p, 78.30.-j

#### Selection rules

- IR: Active modes produce dipole, symmetry operations transforming like x, y, z
- Raman: Active modes produce polarisability, symmetry operations transforming like xy, yz, zx

#### Vibrational spectroscopies

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# Infra-red spectroscopy

- Dispersion of light at frequencies comparable with those of vibrational modes
- Coupling of vibrational with electromagnetic modes
- Central quantities:
  - Dielectric function  $\varepsilon(\omega)$ ,
  - $\varepsilon_1 = \Re{\{\varepsilon(\omega)\}},$
  - $\varepsilon_2 = \Im\{\varepsilon(\omega)\},$
  - Energy-loss  $-\Im\left\{\frac{1}{\varepsilon(\omega)}\right\}$

# IR: System in an electric field

- ullet Displacement from equilibrium position  $u_{I\mu}$
- Equation of motion:

$$M_I \ddot{u}_{I\mu} = -\sum_{J\lambda} \sqrt{M_I M_J} \, D_{I\mu,J\lambda} u_{J\lambda} + \mathbf{Z}^*_{I\mu} \cdot \mathbf{E} \; ,$$

#### where

- dynamical matrix  $D_{l\mu,J\lambda} = \frac{1}{\sqrt{M_l M_J}} \left. \frac{\partial^2 E_{\text{tot}}}{\partial u_{l\mu} \partial u_{J\lambda}} \right|_{u_{l\mu}=0,u_{J\lambda}=0}$  with eigenfrequencies  $\omega_n$  and normalised eigenmodes  $e^n_{l\mu}$
- Born effective charges  $\mathbf{Z}_{l\mu\lambda}^* = -\left. \frac{\partial^2 E_{\mathrm{tot}}}{\partial u_{l\mu}\partial \mathbf{E}_{\lambda}} \right|_{u_{l\mu}=0,\mathbf{E}_{\lambda}=0}$

### IR: Oscillating electric field

- Oscillating field  $\mathbf{E} = \mathbf{E}^0 e^{(i\omega + \eta)t}$
- Linear order:  $u_{I\mu}(t) = u_{I\mu}^0 e^{(i\omega + \eta)t}$

$$\Rightarrow u_{I\mu}^0 = \sum_{nJ\lambda} \frac{1}{(\eta - i\omega)^2 - \omega_n} \frac{e_{I\mu}^n}{\sqrt{M_I}} \frac{e_{J\lambda}^n}{\sqrt{M_J}} \mathbf{Z}_{J\lambda}^* \cdot \mathbf{E}^0$$

 $\bullet$  Thus absorption occurs at eigenfrequencies of the system, with amplitudes  $\propto \textbf{Z}^*$ 

# IR: Intensity from the dipole

• The effective dipole =  $\mathbf{Z}_{I}^{*} \cdot u_{I}$ , and the intensity

$$I_{
m IR}^
u = \sum_\mu \left| \sum_l {f Z}_{l\mu}^* \cdot u_l^
u
ight|^2$$

$$\mathbf{Z}_{I\mu\lambda}^* = -\left.\frac{\partial^2 E_{\text{tot}}}{\partial u_{I\mu} \partial \mathbf{E}_{\lambda}}\right|_{u_{I\mu} = 0, \mathbf{E}_{\lambda} = 0} = \left.\Omega \frac{\partial P}{\partial u_{I\mu}}\right|_{u_{I\mu} = 0} = \left.\frac{\partial F_I}{\partial \mathbf{E}_{\lambda}}\right|_{\mathbf{E}_{\lambda} = 0}$$

(Could be used with finite differences)

#### IR: Z\* from DFT

- Born-Oppenheimer approximation
- (Kohn-Sham) Hamiltonian with a static electric field:

$$\hat{\mathcal{H}}(\mathbf{E}) = \hat{\mathcal{H}}(\mathbf{E} = 0) + \hat{\mathcal{V}}^{el}(\mathbf{E}) + \hat{\mathcal{V}}^{ion}(\mathbf{E}) \; ,$$

where

- $\hat{\mathcal{V}}^{\text{el}}(\mathbf{E}) = -e\,\mathbf{E}\cdot\hat{\mathbf{r}}$
- $\hat{\mathcal{V}}^{\text{ion}}(\hat{\mathbf{E}}) = e \sum_{l} Z_{l} \mathbf{E} \cdot \mathbf{R}_{l}$
- Isolated system:
  - $\mathbf{P}^{\mathrm{el}}(\mathbf{E}) = \frac{e}{\Omega} \int n(\mathbf{r}) \,\hat{\mathbf{r}} \, d\mathbf{r}$
  - $\mathbf{P}^{\text{ion}}(\mathbf{E}) = -e \sum_{l} Z_{l} \mathbf{R}_{l}$

$$\Rightarrow \mathbf{Z}_{I\mu,\lambda}^* = -e\sum_{\mathbf{v}} \frac{\partial}{\partial u_{I\mu}} \langle \psi_{\mathbf{v}} \mid \hat{\mathbf{r}}_{\lambda} \mid \psi_{\mathbf{v}} \rangle + e\sum_{I} Z_{I} \delta_{\mu\lambda}$$

#### IR: Z\* from DFT

• Using DFPT in linear response regime

$$\mathbf{Z}_{l\mu,\lambda}^{*} = -e \sum_{\mathbf{v}} \left( \left\langle \frac{\partial \psi_{\mathbf{v}}}{\partial u_{l\mu}} \mid \hat{\mathbf{r}}_{\lambda} \mid \psi_{\mathbf{v}} \right\rangle + \text{c.c.} \right) + e \sum_{l} Z_{l} \, \delta_{\mu\lambda}$$

### IR: Position operator in extended system

- $\langle \psi_m \mid \mathbf{r} \mid \psi_{\mathbf{v}} \rangle$  not well defined
- Recasting

$$\langle \psi_m \mid \mathbf{r} \mid \psi_{\mathbf{v}} \rangle = \frac{\langle \psi_m \mid [H_{KS}, \mathbf{r}] \mid \psi_{\mathbf{v}} \rangle}{\varepsilon_m - \varepsilon_{\mathbf{v}}} \quad \forall m \neq n ,$$

where 
$$[H_{\rm KS},{\bf r}]=-rac{\hbar^2}{m_{\rm e}}rac{\partial}{\partial{\bf r}}+[V_{\rm NL},{\bf r}]$$

Using Green's functions

$$(H_{\mathrm{KS}} - \varepsilon_{\nu}) \left| \overline{\psi}_{\nu}^{\lambda} \right\rangle = P_{m} \left[ H_{\mathrm{KS}}, \mathbf{r} \right] \left| \psi_{\nu} \right\rangle ,$$

where 
$$\left|\overline{\psi}_{\mathbf{v}}^{\lambda}\right>=P_{m}\,\mathbf{r}_{\lambda}\left|\psi_{\mathbf{v}}\right>$$

solve\_e.f90

### IR: Born effective charges

$$\mathbf{Z}_{I\mu,\lambda}^{*} = 2e\sum_{\mathbf{v}} \left( \left\langle \frac{\partial \psi_{\mathbf{v}}}{\partial u_{I\mu}} \mid \overline{\psi}_{\mathbf{v}}^{\lambda} \right\rangle + \mathrm{c.c.} \right) + e\sum_{I} Z_{I} \delta_{\mu\lambda}$$
zstar eu.f90

#### IR: Dielectric matrix and polarisability

As by-products we obtain

$$\varepsilon_{\mu\lambda}^{\infty} = \delta_{\mu\lambda} + 4\pi \frac{\partial P_{\lambda}}{\partial \mathbf{E}_{\mu}} = \delta_{\mu\lambda} - \frac{4\pi}{\Omega} \frac{\partial^{2} \mathbf{E}_{\text{tot}}}{\partial \mathbf{E}_{\lambda} \partial \mathbf{E}_{\mu}} ,$$

where 
$$P_{\lambda}=-rac{1}{\Omega}rac{\partial \mathcal{E}_{\mathrm{tot}}}{\partial \mathbf{E}_{\lambda}}$$
, and  $\alpha_{\mu\lambda}$ 

# IR: Dielectric response

#### Also

$$\varepsilon_{\mu\lambda}^{1} = \varepsilon_{\mu\lambda}^{\infty} - \frac{4\pi}{\Omega} \sum_{\nu} \frac{F_{\mu}^{\nu} F_{\lambda}^{\nu}}{\omega^{2} - \omega_{\nu}^{2}}$$

$$\varepsilon_{\mu\lambda}^{2} = \frac{4\pi}{\Omega} \sum_{\nu} \frac{F_{\mu}^{\nu} F_{\lambda}^{\nu}}{2\omega_{\nu}^{2}} \delta(\omega - \omega_{\nu})$$

where 
$$F^{
u}_{\mu} = \sum_{l\lambda} \mathbf{Z}^*_{l\mu,\lambda} rac{u_{l\lambda}}{\sqrt{M_l}}$$

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#### Raman spectroscopy in QE: Implementation

VOLUME 90, NUMBER 3

PHYSICAL REVIEW LETTERS

week ending 24 JANUARY 2003

#### First-Principles Calculation of Vibrational Raman Spectra in Large Systems: Signature of Small Rings in Crystalline SiO<sub>2</sub>

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We present an approach for the efficient calculation of vibrational Raman intensities in periodic systems within density functional theory. The Raman intensities are computed from the second orderivative of the electronic density matrix with respect to a uniform electric field. In contrast to previous approaches, the computational effort required by our method for the evaluation of the intensities is negligible compared to that required for the calculation of vibrational frequencies. As a first application, we study the signature of 3 and 4-membered rings in the Raman spectra of several polymorphs of SiO<sub>2</sub>, including a zeolite (H-ZSM-18) having 102 atoms per unit cell.

DOI: 10.1103/PhysRevLett.90.036401

PACS numbers: 71.15.Mb, 78.30.-j

#### Raman spectroscopy: Intensity

• Intensity of eigenmode  $\mu$ :

$$I^{
u} \propto rac{\left(\omega_{i}-\omega_{
u}
ight)^{4}}{\omega_{
u}}\left|\mathbf{e}_{i}\cdot\overline{\overline{A}}^{\,
u}\cdot\mathbf{e}_{s}
ight|^{2}\left(n_{
u}+1
ight)\,,$$

#### where

•  $\mathbf{e}_i$ ,  $\mathbf{e}_s$  = polarisation of incident, scattered photon,  $n_{\nu}$  = Bose-Einstein occupation factor and

$$\overline{\overline{A}}^{\nu} = \sum_{l\mu} \frac{\partial^3 E^{\mathrm{el}}}{\partial \mathbf{E} \, \partial \mathbf{E} \, \partial \mathbf{U}_{l\mu}} \frac{\mathbf{e}_{l\mu}}{\sqrt{M_l}} \; ,$$

or

$$\overline{\overline{A}}^{\nu} = \sum_{l_{\prime\prime}} \frac{\partial \overline{\alpha}}{\partial u_{l\mu}} \frac{e_{l\mu}}{\sqrt{M_{l}}} ,$$

the change in polarisability along a displacement along the normal mode

#### Raman spectroscopy: Implementation

 An alternative formulation, using the Kohn-Sham density matrix

$$\begin{split} n(\mathbf{r},\mathbf{r}') &= \sum_{\mathbf{v}} |\psi_{\mathbf{v}}\rangle \, \langle \psi_{\mathbf{v}}| \ , \\ \frac{\partial E^{\mathrm{el}}}{\partial u_{l\mu}} &= 2\mathrm{Tr} \left[ n(\mathbf{r},\mathbf{r}') \frac{\partial V^{\mathrm{ext}}}{\partial u_{l\mu}} \right] \ , \ (\mathrm{H}-\mathrm{F} \ \mathrm{theorem}) \\ \frac{\partial^3 E^{\mathrm{el}}}{\partial \mathbf{E} \, \partial \mathbf{E} \, \partial u_{l\mu}} &= 2\mathrm{Tr} \left[ \frac{\partial^2 n(\mathbf{r},\mathbf{r}')}{\partial \mathbf{E} \, \partial \mathbf{E}} \frac{\partial V^{\mathrm{ext}}}{\partial u_{l\mu}} \right] \end{split}$$

Evaluation of \(\frac{\partial^2 n(\mathbf{r},\mathbf{r}')}{\partial \mathbf{E} \partial \mathbf{E}}\) requires six second-order calculations, independent of number of atoms

#### Raman spectroscopy: Implementation

Perturbation theory:

$$\frac{\partial^{2} n(\mathbf{r}, \mathbf{r}')}{\partial \lambda \partial \mu} = \sum_{\mathbf{v}} \left[ \left| P \eta_{\mathbf{v}}^{(\lambda, \mu)} \right\rangle \langle \psi_{\mathbf{v}} \right| + \left| P \frac{\partial \psi_{\mathbf{v}}}{\partial \lambda} \right\rangle \left\langle \frac{\partial \psi_{\mathbf{v}}}{\partial \mu} P \right| + \sum_{\mathbf{v}'} \left| \psi_{\mathbf{v}'} \right\rangle \left\langle \frac{\partial \psi_{\mathbf{v}'}}{\partial \lambda} P \right| P \frac{\partial \psi_{\mathbf{v}}}{\partial \mu} \right\rangle \langle \psi_{\mathbf{v}} | + \text{c.c.}$$

- Terms  $\left|P\frac{\partial\psi_{\nu}}{\partial\lambda}\right\rangle$ ,  $\left|P\eta_{\nu}^{(\lambda,\mu)}\right\rangle$  calculated using Green's functions
- Involve  $\partial V^{\text{KS}}/\partial \lambda$ ,  $\partial^2 V^{\text{KS}}/(\partial \lambda \partial \mu)$ , self-consistency needed

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# Usage: IR, input

```
Normal modes for CO2

&inputph

tr2_ph = le-14

prefix = 'CO2'

amass(1) = 12.010

amass(2) = 15.999

outdir = './scratch/'

epsil = .true.

trans = .true.

asr = .true.

fildyn = 'dmat.co2'

//
```

# Usage: IR, output

```
Electric Fields Calculation

iter # 1 total cpu time : 81.9 secs av.it.: 6.7
    thresh= 1.000E-02 alpha_mix = 0.700 |ddv_scf|^2 = 2.125E-08

iter # 2 total cpu time : 113.7 secs av.it.: 14.0
    thresh= 1.458E-05 alpha_mix = 0.700 |ddv_scf|^2 = 7.433E-08

...

iter # 7 total cpu time : 261.3 secs av.it.: 13.7
    thresh= 1.306E-08 alpha_mix = 0.700 |ddv_scf|^2 = 1.010E-14

iter # 8 total cpu time : 291.5 secs av.it.: 14.0
    thresh= 1.005E-08 alpha_mix = 0.700 |ddv_scf|^2 = 4.752E-17

End of electric fields calculation
```

#### Usage: IR, output

```
Dielectric constant in cartesian axis
            1.097029260
                              0.034059205
                                               0.000000675)
            0.034059205
                              1.097029260
                                               0.000000675)
            0.000003118
                              0.000003118
                                               1.063644537 )
Polarizability (a.u.) ^3
                                          Polarizability (A^3)
20.52
          7.44
                    0.00
                                    3.0413
                                                  1.1021
                                                                0.0001
7.44
         20.52
                  0.00
                                    1.1021
                                                  3.0413
                                                                0.0001
0.00
        0.00
                  13.61
                                     0.0000
                                                  0.0000
                                                                2.0166
    Effective charges (d Force / dE) in cartesian axis
     atom
              1 C
            1.36251
                            0.91927
                                           0.00000)
 Ex
            0.91927
                           1.36251
                                          0.00000)
 Εv
            -0.00015
                           -0.00015
                                           0.44133 )
 Εz
     atom
               2 0
            -0.67618
                           -0.46117
                                          -0.00021)
 Ex
 Ey
           -0.46117
                           -0.67618
                                         -0.00021 )
 Εz
           -0.00032
                           -0.00032
                                         -0.22308)
               3 0
     atom
            -0.67538
                           -0.46058
                                         -0.00009)
 Ex
           -0.46058
                           -0.67538
                                         -0.00009)
 Ey
            0.00009
                           0.00009
                                         -0.22323)
 Εz
```

# Usage: Raman, input

```
&inputph
   tr2 ph
                   = 1e-14
   prefix
                   = ' qar - w 03 - 1H'
   amass(1)
                   = 12.0107
               = 1.0079
   amass(2)
                = './scratch/'
   outdir
                = 'gar-w_03-1H-ph.dyn'
   fildyn
   recover
                   = .false.
   epsil
                  = .true.
   lraman
                 = .true.
                 = .true.
   trans
   fildrho = "gar-w_03-1H-ph.fildrho"
   max seconds
                   = 350000
0.0 0.0 0.0
```

### Usage: Raman, output

```
Raman tensor (au^-1) in cartesian axis
 atom
        0.664735398
                         -0.007862947
                                             0.0000000000)
       -0.007862947
                          0.011455170
                                             0.000000000)
       0.000000000
                          0.000000000
                                             0.000420683)
       0.109910490
                          0.045691031
                                             0.000000000)
       0.045691031
                         -0.053438482
                                             0.000000000)
        0.000000000
                          0.000000000
                                            -0.003141788)
       -0.000000000
                          0.000000000
                                            -0.031312700 )
        0.000000000
                         -0.000000000
                                            -0.001689833 )
       -0.031312700
                         -0.001689833
                                            -0.000000000
 atom
```

. . .

### Usage: Raman, output

```
Raman tensor (A^2)
atom #
        1
              pol.
0.408888389268E+02
                       -0.483661299120E+00
                                                0.00000000000E+00
-0.483661299120E+00
                        0.704624143907E+00
                                                0.0000000000E+00
0.00000000000E+00
                        0.00000000000E+00
                                                0.258768276476E-01
              pol.
atom #
0.676075372141E+01
                        0.281052160573E+01
                                                0.00000000000E+00
0.281052160573E+01
                       -0.328707859334E+01
                                                0.0000000000E+00
0.00000000000E+00
                        0.00000000000E+00
                                               -0.193255963525E+00
atom # 1
              pol.
-0.277499384006E-19
                        0.00000000000E+00
                                               -0.192608963659E+01
0.00000000000E+00
                       -0.170828783391E-22
                                               -0.103944090539E+00
-0.192608963659E+01
                       -0.103944090539E+00
                                               -0 192182381315E-23
atom # 2
              pol.
-0.408888389268E+02
                       -0.483661299120E+00
                                                0.0000000000E+00
-0.483661299120E+00
                       -0.704624143907E+00
                                                0.00000000000E+00
 0.00000000000E+00
                        0.00000000000E+00
                                               -0.258768276476E-01
```

#### Usage: Symmetries

```
Mode symmetry, D 2h (mmm) point group:
freq (1 - 1) =
                   -43.9 [cm-1] --> B_1u
freq (2 - 2) =
                   24.8 [cm-1]
                                 --> B 3q
freq (3 - 3) =
                   47.4 [cm-1] --> B 3u
freq (4 - 4) = 73.1 [cm-1] --> B_2u
freq (5 - 5) =
                    173.8 [cm-1] --> B 1u
                    342.2 [cm-1] --> A_u
freq (6 - 6) =
freq (7 - 7) =
                    350.7 [cm-1] --> B_1q
                                                  R
freq ( 8 - 8) =
                    353.0 [cm-1]
                                 --> B_3q
                                                  R
freq (9 - 9) =
                    386.3 [cm-1]
                                  --> B 2q
                                                  R
freq ( 10 - 10) =
                    530.4 [cm-1]
                                 --> B 1u
freq (11 - 11) =
                    536.8 [cm-1]
                                 --> A a
freq (12 - 12) =
                    540.1 [cm-1]
                                 --> B 1q
freq (13 - 13) =
                    560.0 [cm-1] --> B_2g
freq (14 - 14) =
                    651.0 [cm-1]
                                 --> B 3u
freq (15 - 15) =
                    730.4 [cm-1]
                                 --> B 1u
freq (16 - 16) =
                    732.3 [cm-1] --> A u
freq (17 - 17) =
                    740.5 [cm-1]
                                 --> B 3α
freq ( 18 - 18) =
                    762.7 [cm-1] --> B_3u
                                                  Т
freq (19 - 19) =
                    826.2 [cm-1] --> B 2u
freq (20 - 20) =
                    846.5 [cm-1] --> B 2g
                  912.0 [cm-1] --> B 2q
freq ( 21 - 21) =
freq (22 - 22) =
                    912.4 [cm-1]
                                 --> A u
```

#### Usage

#### Character table for point group D<sub>2h</sub>

$D_{2h}$	E	$C_2(z)$	C <sub>2</sub> (y)	C <sub>2</sub> (x)	i	$\sigma(xy)$	$\sigma(xz)$	$\sigma(yz)$	linear functions, rotations	quadratic functions	cubic functions
$A_g$	+1	+1	+1	+1	+1	+1	+1	+1	-	$x^2, y^2, z^2$	-
$B_{1g}$	+1	+1	-1	-1	+1	+1	-1	-1	$R_z$	xy	-
$B_{2g}$	+1	-1	+1	-1	+1	-1	+1	-1	$R_{y}$	XZ	-
$B_{3g}$	+1	-1	-1	+1	+1	-1	-1	+1	$R_{x}$	yz	-
$A_u$	+1	+1	+1	+1	-1	-1	-1	-1	-	-	xyz
$B_{1u}$	+1	+1	-1	-1	-1	-1	+1	+1	z	-	$z^3$ , $y^2z$ , $xz^2$
$B_{2u}$	+1	-1	+1	-1	-1	+1	-1	+1	у	-	$yz^2$ , $x^2y$ , $y^3$
$B_{3u}$	+1	-1	-1	+1	-1	+1	+1	-1	x	-	$xz^2$ , $xy^2$ , $x^3$

#### Usage: Symmetries

```
Mode symmetry, D 2h (mmm) point group:
freq (1 - 1) =
                   -43.9 [cm-1] --> B_1u
freq (2 - 2) =
                   24.8 [cm-1]
                                 --> B 3q
freq (3 - 3) =
                   47.4 [cm-1] --> B 3u
freq (4 - 4) = 73.1 [cm-1] --> B_2u
freq (5 - 5) =
                    173.8 [cm-1] --> B 1u
                    342.2 [cm-1] --> A_u
freq (6 - 6) =
freq (7 - 7) =
                    350.7 [cm-1] --> B_1q
                                                  R
freq ( 8 - 8) =
                    353.0 [cm-1]
                                 --> B_3q
                                                  R
freq (9 - 9) =
                    386.3 [cm-1]
                                  --> B 2q
                                                  R
freq ( 10 - 10) =
                    530.4 [cm-1]
                                 --> B 1u
freq (11 - 11) =
                    536.8 [cm-1]
                                 --> A a
freq (12 - 12) =
                    540.1 [cm-1]
                                 --> B 1q
freq (13 - 13) =
                    560.0 [cm-1] --> B_2g
freq (14 - 14) =
                    651.0 [cm-1]
                                 --> B 3u
freq (15 - 15) =
                    730.4 [cm-1]
                                 --> B 1u
freq (16 - 16) =
                    732.3 [cm-1] --> A u
freq (17 - 17) =
                    740.5 [cm-1]
                                 --> B 3α
freq ( 18 - 18) =
                    762.7 [cm-1] --> B_3u
                                                  Т
freq (19 - 19) =
                    826.2 [cm-1] --> B 2u
freq (20 - 20) =
                    846.5 [cm-1] --> B 2g
                  912.0 [cm-1] --> B 2q
freq ( 21 - 21) =
freq (22 - 22) =
                    912.4 [cm-1]
                                 --> A u
```

# Usage: Extracting intensities, plotting spectra

- Here we use dynmat.x, a simplified version of analysis
- Limitations: Beyond LDA only with shortcuts
- No ultra-soft PP, PAW with electric field