Problem sheet : 2

PHY 202; Relativity and quantum physics.
IISER, Pune. (April, 2009)

This is not meant for evaluation and need not be submitted back. However, you are welcome to approach me for any doubts and clarifications.

1. A wavefunction is given by $\phi(x) = A\ e^{-x/2}$ for $x \in [0, \infty]$. Determine the normalisation constant A.

2. For the problem in (1), calculate the probability for the particle to lie in the range $1 \leq x \leq \infty$.

3. Find the classical turning points for a particle with energy $E = mv^2/2 + bx$ $(b > 0)$.

4. Find the mean position $\langle x \rangle$ for a particle in a infinite potential well of width L.

5. Determine $\langle x^2 \rangle$ for a particle in a infinite potential well of width L.

6. What are physical meanings of three quantum numbers n, l, m_l in the quantum hydrogen atom problem.

7. Write down the volume element dV in spherical polar coordinate system. For $1s$ state of hydrogen atom, show by integration over all the r, θ, ϕ variables that the probability of finding the electron somewhere in space is one.

8. List the sets of quantum numbers possible for $n = 4$ hydrogen atom.

9. Find the most probable values of r at which electron is likely to be present in the $2s$ state of hydrogen atom.

10. Show that the most probable value of r for $1s$ state of hydrogen atom is the Bohr radius a_0.

11. Find the uncertainty Δr in r for $1s$ state of hydrogen atom.

12. Calculate the probability of finding a $1s$ electron in a hydrogen atom at a distance greater than a_0 from the nucleus.

13. Hydrogen atomic states are given by $\psi_{n,l,m_l}(r, \theta, \phi)$. Calculate the integral $\int \psi_{1,0,0}(r, \theta, \phi)\ \psi_{2,0,0}(r, \theta, \phi)\ d\theta\ d\phi\ dr$. This is one special case of general orthogonality relation in quantum mechanics.

14. Show that $\Theta_{20}(\theta) = \sqrt{10}/4\ (3\cos^2\theta - 1)$ is already normalised.

15. Consider a finite potential well of height V_0. Set up the Schrodinger equation for a particle with energy $E < V_0$ inside and outside the well. Write down the admissible solutions for each region, i.e, inside and outside the well.

16. Hydrogen atom emits a radiation of wavelength 102.55 nm while returning to ground state. Find the quantum number of the orbit from which electron returned to the ground state.

Hydrogen atom wavefunctions :

Wavefunctions shown below represent the standard notation $\psi_{n,l,m_l}(r, \theta, \phi)$. For some more hydrogen atom states, see page 206 of Arthur Beiser's book.

$$\psi_{1,0,0}(r, \theta, \phi) = \frac{1}{\sqrt{\pi} a_0^{3/2}}\ e^{-r/a_0}$$

$$\psi_{2,0,0}(r,\theta,\phi) = \frac{1}{4\sqrt{2\pi}a_0^{3/2}}\left(2 - \frac{r}{a_0}\right) e^{-r/2a_0}$$

$$\psi_{2,1,0}(r,\theta,\phi) = \frac{r\cos\theta}{4\sqrt{2\pi}a_0^{5/2}}\, e^{-r/2a_0}$$

Some useful constants :

$c = 2.997 \times 10^{8}$ m/s (speed of light)
$\hbar = 1.054 \times 10^{-34}$ J.s (Planck's constant/2π)
$m = 9.109 \times 10^{-31}$ kg (Electron mass)
$k = 1.380 \times 10^{-23}$ J/K (Boltzmann constant)
$a_0 = 0.529 \times 10^{-10}$ m (Bohr radius)