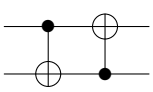
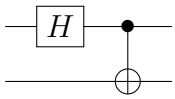
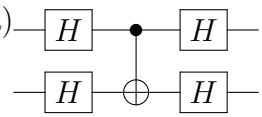
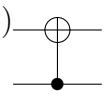
PH441 / 6414 QUANTUM INFORMATION - PROBLEM SET 1

These are sample problems for you to try out. You can also try problems from text books.

1) Obtain unitary transformations corresponding to the following two-qubit circuits

(a)  (b) 

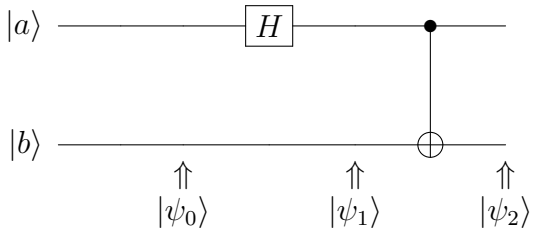
2) Show that the following two circuits are equivalent by working out their action on $|00\rangle$.

(a)  (b) 

3) Obtain the matrix form of X gate in the basis of $|\alpha\rangle$ and $|\beta\rangle$

$|\alpha\rangle = \frac{|0\rangle+\sqrt{2}|1\rangle}{\sqrt{3}}$ and $|\beta\rangle = \frac{|0\rangle-\sqrt{2}|1\rangle}{\sqrt{3}}$.

4) Consider the following circuit give below:



If $|a\rangle = \alpha_0|0\rangle + \beta_0|1\rangle$ and $|b\rangle = \alpha_0|0\rangle + \beta|1\rangle$, obtain explicit form for $|\psi_0\rangle, |\psi_1\rangle$ and $|\psi_2\rangle$.

5) In problem (4), find the probability of a measurement giving $|0\rangle$ at the final state.

6) Obtain the matrix form of NOT gate in Hadamard basis.

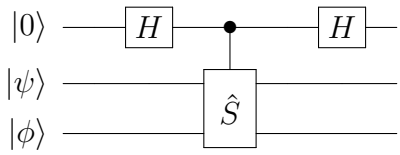
7) Consider the state

$$|\psi\rangle = \frac{2}{3}|00\rangle + \frac{1}{3}|01\rangle - \frac{2}{3}|11\rangle.$$

Let $|\varphi\rangle = (I \otimes \hat{H})|\psi\rangle$. Write $|\varphi\rangle$ in computational basis.

8) What is the gate represented by $\hat{H}\hat{X}\hat{H}$?

9) Suppose $\hat{S}$ represents an operator to perform SWAP operation such that $\hat{S}|x\rangle|y\rangle = |y\rangle|x\rangle$, $x, y \in \{0, 1\}$. Find the output of following quantum circuit



10) Show that if $\hat{U}$ and $\hat{V}$ are unitary operators, then $\hat{U} \otimes \hat{V}$ is also unitary.

11) Write the matrix form of 3 qubit Toffoli gate.

12) Write the state $(\hat{I} \otimes \hat{H}) |\beta_{00}\rangle$ in computational basis.