Indian Institute of Science Education and Research Pune
Mid-semester Exam, January (2024) semester.

Course name: Nonlinear Dynamics **Course code:** PH 4273 / 6423
Date: 23.2.2024 (3:00 PM to 5:00 PM) *Duration:* 2 hours
Instructor : M. S. Santhanam Maximum marks: 45

- Attempt any one among the questions 1 and 2.
- Answer all the questions from 3 to 6.
- This question paper has 6 questions. See the reverse side as well.
- If you draw sketches as an answer, label the axes. No marks without axes labels.
- SHOW ALL THE STEPS CLEARLY in your calculations while arriving at an answer.
- Use the same symbols and notation given in the question. Do not use your own.

1.(a) For the dynamical system $\dot{x} = 1 - 2\cos(x/2)$, sketch the state space, flowline and determine the fixed point and show them in your figure.
(b) For $\dot{x} = 1 - e^{-x^2}$, find the fixed points and its stability by any method of your choice.
(3+2)

2.(a) Let a dynamical system be described by a second order ordinary differential equation $\ddot{x} + \alpha\dot{x} + f(x) = 0$, where α is a parameter. In this case, briefly explain why trajectories cannot cross in state space.
(b) What are the differences (if any) between a periodic orbit and a limit cycle oscillation ? Explain your answer.
(c) Give an example (by writing its equation of motion) of any system that would show linear or nonlinear center. (2+2+1)

3. For a conservative system (energy E is conserved) given by the equation of motion

$$\ddot{x} + 4\alpha x^3 = 0,$$

where α is a constant, find the time period of oscillation. Sketch the time period as a function of energy E of the system. (7+3)

4. Consider the system

$$\ddot{x} + 3\dot{x} - 2(x-1)^2 = 0.$$

(a) Linearise this system and find the Jacobian matrix for a general fixed point.
(b) Use the result in (a) to find the stability of the fixed point of this system.
(c) Using the information in (a) and (b), plot the phase portrait. (3+3+4)

5. For the system with α as a tunable parameter

$$\dot{x} = \alpha + \frac{x}{2} - \frac{x}{1+x},$$

(a) By reducing this to a normal form, determine the type of bifurcation in this system.
(b) Find the critical value of α at which bifurcation takes place.
(c) Sketch the state space just before and just after the bifurcation. (5+2+3)

6. Consider a two-dimensional system given by

$$\begin{aligned} \dot{x} &= -\mu y + x - \alpha x(x^2 + y^2), \\ \dot{y} &= -\mu x + y - \alpha y(x^2 + y^2). \end{aligned}$$

In this μ and α are parameters. Fixed point for this system is at (0,0).
(a) Determine the stability and type of this fixed point (node, spiral etc. etc.).
(b) Use Poincare-Bendixson theorem to show analytically that a limit cycle exists in this system.
(c) Determine the range of μ and α for which limit cycle exists. (3+3+4)

- You might need this information :

$$\int_{-1}^{1} \frac{dx}{\sqrt{1-x^4}} = \frac{2\sqrt{\pi}\,\Gamma(\frac{5}{4})}{\Gamma(\frac{3}{4})} \approx 2.62206$$