

Time: 50 minutes. Maximum Marks : 20.

For questions involving drawings graphs, label your axes.

Show all steps of the calculations clearly.

1. For the tent map given by

$$x_{n+1} = r \left(1 - 2 \left| x_n - \frac{1}{2} \right| \right),$$

where r is a parameter. Find the fixed point for $x > 1/2$ and its stability. (3)

2. Calculate the Lyapunov exponent of the map : $x_{n+1} = 10x_n \pmod{1}$ with $x \in [0, 1]$. (5)

4. For the doubling map (shift map), how many orbits of period-3 are there. Enumerate all the periodic orbits of period-3. (6)

4. A conjugacy transformation defined by $x_n = ay_n + b$ is applied to the logistic map $x_{n+1} = \alpha x_n(1 - x_n)$. If the map conjugate to the logistic map takes the form

$$u_{n+1} = u_n^2 - C_1 u_n + C_2,$$

then find the constants C_1 and C_2 in terms of α, a and b . (6)