

Time: 50 minutes. Max Marks : 25

*Show all the steps of your calculations, marks will be deducted for missing steps.
For questions involving drawings graphs, label your axes.
Use the same notation and symbols given in the question.
Assumptions, if any, must be explicitly stated.*

1. Determine and plot the potential function for the following cases and mark the equilibrium points and classify their stability.

(a) $\dot{x} = 1 - x^2$, (b) $\dot{x} = 1 + x - x^3$. (3+3)

2. Consider the system (α is a parameter)

$$\dot{x} = x + \frac{\alpha x}{1 + x^2}.$$

Show that this system displays pitchfork bifurcation. Classify it as supercritical or subcritical. (6)

3. Let $\theta \in (-\pi, \pi)$. Draw phase portrait for

$$\dot{\theta} = \mu + \cos 2\theta$$

for parameter $\mu = 1$. What is the range of μ for which this system displays oscillations in θ . (5)

4. Consider the first order differential equation;

$$\frac{dx}{dt} = ax + bx^3 - cx^5,$$

where a, b, c are the parameters. If x has dimensions of length, and t has dimensions of time, then obtain a dimensionless version of this equation in the following form :

$$\frac{dy}{d\tau} = y + y^3 - Qy^5$$

Find the relation between Q and a, b, c (8)