PHY313 : Assignment 5

IISER, Pune. (November, 2019)

(NOTE : *This is a sample selection of problems. You must try out more problems from other text books as well. Books by R. Shankar, Walter Greiner and Griffiths are a good source of problems. Some of these problems are taken from these books.*)

1. For the most general spinor $psi = a\chi_+ + b\chi_-$ where χ_+ and χ_- are up and down spins respectively, compute the following averages ; $\langle S_x \rangle$, $\langle S_x^2 \rangle$ and similarly for y and z component of spin.

2. An electron is in the spin state $\psi = A \begin{pmatrix} 3i \\ 4 \end{pmatrix}$, determine the normalisation constant, and $\langle S_x \rangle$, $\langle S_x^2 \rangle$. Use this to find the uncertainty in S_x.

3. Prove the identity; $(A.\sigma)(B.\sigma) = A.BI + i(A \times B).\sigma$. In this A and B are commuting operators.

4. Construct matrix $\mathbf{S}_n$ which represents the component of spin angular momentum along an arbitrary direction $\hat{n}$. Use spherical coordinates to specify the arbitrary direction. Find its eigenvalues and eigenvectors.

5. Show that the Pauli matrices are traceless.

6. Do problems 14.3.2 to 14.3.8 from R. Shankar's book.

7. At $t = 0$, an electron is in the spin-up state. A steady magnetic field $\mathbf{B} = B\ \hat{i}$ with $B = 100$ Gauss is turned on. What is the time taken for the spin to flip to spin-down state.

8. An electron is at rest in an oscillating magnetic field given by, $\mathbf{B} = B_0 \cos(\omega t)\ \hat{k}$, where B_0 and ω are constants. If the electron is assumed to start from spin-up state, find the state at any other time t.

9. Consider a system with two states. It can be thought of as up- and down-states of spin. Let the Hamiltonian of such a system be given by, $H = a\left(|1\rangle\langle 1| - |0\rangle\langle 0| + |1\rangle\langle 0| + |0\rangle\langle 1|\right)$. Find the energy eigenvalues and the states in terms of the up- and down-spin states.