PHY313 : Assignment 4

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(NOTE : *This is a sample selection of problems. You must try out more problems from other text books as well. Books by R. Shankar, Walter Greiner and Griffiths are a good source of problems.*)

1. Show that invariance under time translation leads to energy conservation, i.e, $\langle H \rangle = 0$.

2. If a Hamiltonian H is parity invariant, show that $\exp(-iHt/\hbar)$ is also parity invariant.

3. In (x, y, z) coordinate system, angular momentum operator is given by $L_z = XP_y - YP_x$. Perform a coordinate transformation to spherical polar coordinate system and show that $L_z \to -i\hbar\frac{\partial}{\partial\phi}$.

4. Starting from $U[R(-\epsilon_z\mathbf{k})]T(-\epsilon)U[R(\epsilon_z\mathbf{k})]T(\epsilon)$, show that $[P_y, Lz] = i\hbar P_z$ and $[P_x, Lz] = -i\hbar P_y$. In this, U and T are rotation and translation operators.

5. Do the problems 12.3.3 and 12.3.4 given in R. Shankar's book.

6. Solve the eigenvalue problem of a particle constrained to move on a circle of radius a.

7. A series of classical rotations leads to the following operation;

$$R(-\epsilon_y\mathbf{j})R(-\epsilon_x\mathbf{i})R(\epsilon_y\mathbf{j})R(-\epsilon_x\mathbf{i}) = R(-\epsilon_x\epsilon_y\mathbf{k})$$

From this, deduce that $[L_i, L_j] = i\hbar\sum_{k=1}^{3}\epsilon_{123}L_k$, where $i, j = 1, 2, 3$.

8. Do problems 12.5.2 and 12.5.3 from R. Shankar's book.

9. If $H = p^2/2m + V(r)$, then show that the Hamiltonian commutes with all three components of angular momentum operator.

10. If $Y_2^1(\theta, \phi) = -\sqrt{15/8\pi}\sin\theta\cos\theta\; e^{i\phi}$, find $Y_2^2(\theta, \phi)$.

11. Do problems 12.5.11 to 12.5.14 from R. Shankar's book.

12. Find the energy eigenvalues and eigenfunctions of the isotropic harmonic oscillator in three dimensions.

13. Do problems 13.1.1 to 13.1.4 from R. Shankar's book.

14. Show that probability to find the electron is maximum at $r = N^2a_0$, where a_0 is the Bohr radius and N is the principal quantum number.

15. Show that for a Hamiltonian $H = p^2/2m + V(x)$, translation invariance is always valid.

16. Deduce the parity property of the spherical harmonics, $Y_l^m(\theta, \phi)$.

17. Find the energy levels of a particle of mass μ in the potential $V(r) = a/r^2 + br^2$, where $a > 0$ and $b > 0$ are constants.

18. Obtain the degree of degeneracy for the hydrogen atom problem.

19. In the hydrogen atom problem, show that probability current $J > 0$ only for the azimuthal direction.

20. Construct the wavefunction $\psi(r, \theta, \phi)$ for the hydrogen atom in the state $N = 4, l = 3, m = 3$. Use this state to find $\langle r \rangle$.

21. Solve the eigenvalue problem for the operators L^2 and L_z, both of which commute with the Hamiltonian H.