PHY313 : Assignment 3

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(NOTE : *This is a sample selection of problems. You are encouraged to try out more problems from other text books as well. Books by R. Shankar, Walter Greiner and Griffiths are a good source of problems.*)

1. Compute the following matrix elements and show that
(a) $\langle n|x^4|n\rangle = (3 + 6n(n-1))\hbar^2/4m^2\omega^2$,
(b) $\langle 0|x^3|0\rangle = 9\sqrt{3\hbar/2m\omega}$
(c) $\langle 0|p^2|0\rangle = 3\hbar^2/4m^2\omega^2$

2. Estimate the ground state energy of the standard harmonic oscillator using the Heisenberg's uncertainty principle $\Delta x \Delta p \geq \hbar/2$.

3. Consider a particle in potential given by, $V(x) = \frac{1}{2}m\omega^2 x^2$ for $x > 0$ and $V(x) = 0$ for $x \leq 0$. Find the eigenvalues and eigenfunctions.

4. Use the raising and lowering operators of the harmonic oscillator problem to obtain explicit matrix form for $\widehat{X}, \widehat{P}$ and $\widehat{X}^2 + \widehat{P}^2$.

5. Show that $\langle a(t)\rangle = e^{i\omega t}\langle a(0)\rangle$ and $\langle a^\dagger(t)\rangle = e^{i\omega t}\langle a^\dagger(0)\rangle$.

6. Project $a|0\rangle = 0$ in momentum basis and obtain the ground state of harmonic oscillator in momentum representation.

7. At $t = 0$, a one-dimensional harmonic oscillator is in the state $\psi(t = 0) = \sqrt{\frac{3}{4}}u_0 + i\sqrt{\frac{1}{4}}u_1$. Calculate the expected value of p as a function of time.

8. Compute the following commutators; (i) $[\widehat{N}, \widehat{a}]$ and (ii) $[\widehat{N}, \widehat{a}^\dagger]$

9. Find the expectation value of energy of the state $a^\dagger|0\rangle$.

10. Normalise the coherent state given by,

$$|\alpha\rangle = \sum_n \frac{\alpha^n}{\sqrt{n!}}|n\rangle.$$

11. Obtain the coordinate representation of the normalised coherent state given in problem 10. That is, let $\langle x|\alpha\rangle = \psi_\alpha(x)$, and find $\psi_\alpha(x)$.

12. By using the commutation relation $[a, a^\dagger] = 1$, show that

$$e^{-\beta a^\dagger} a\, e^{\beta a^\dagger} = \beta + a$$

13. Use the result of problem 12 to show that $|\beta\rangle = Ne^{\beta a^\dagger}|0\rangle$ (where N is normalisation constant) is a coherent state that satisfies $a|\beta\rangle = \beta|\beta\rangle$.

14. If $|\psi_\epsilon\rangle$ represents a state translated by ϵ, then assume translational invariance for Hamiltonian H to show that linear momentum is conserved.

15. Show that for a Hamiltonian $H = p^2/2m + V(x)$, translation invariance is always valid.

16. Show that the eigenstates of harmonic oscillator are also the eigenstates of parity operator

17. Consider the two dimensional harmonic oscillator, whose Hamiltonian is given by,

$$H = \frac{p_x^2}{2m} + \frac{p_x^2}{2m} + \frac{1}{2}m\omega^2(x^2 + y^2)$$

Show that quantum energy levels are given by $E = (2k + |m| + 1)\hbar\omega$, where $k = 0, 1, 2, \ldots$ and m is the magnetic quantum number associated with L_z. Note that you have to transform the Hamiltonian to polar coordinates. For hints, see the steps mentioned in chapter 12 (section 12.3) of Shankar's book.

18. For a periodic potential $V(x)$ being a series of delta functions spaced d apart, show that the spectra displays forbidden and allowed energy bands.

19. Let a Bloch wavefunction be $\phi(x) = e^{ikx}u(x)$, where $u(x)$ is periodic. What is the period of Bloch wavefunction if (i) $kd = 2l\pi$ and (ii) $kd = (2l + 1)\pi$. In this, d is periodicity of the potential.