PHY321: Assignment 2

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(NOTE : *This is a sample selection of problems. You are encouraged to try out more problems from other text books as well. Books by R. Shankar and Walter Greiner are a good source of problems.*)

1. The classical Hamiltonian for the harmonic oscillator is,

$$H = \frac{p^2}{2m} + \frac{m\omega^2 x^2}{2}.$$

Obtain the Hamiltonian operator in momentum representation. Derive every step of the calculation.

2. Calculate the matrix elements of $\langle n|X^2|m\rangle$ where $|n\rangle$ and $|m\rangle$ are harmonic oscillator states.

3. Calculate $\langle n|P^2|m\rangle$ and $\langle n|P|m\rangle$ where $|n\rangle$ and $|m\rangle$ are harmonic oscillator states.

4. Consider the ground state of the harmonic oscillator. Determine this state in the momentum representation.

5. Using the formalism of raising and lowering operators, a and $a^\dagger$, find the ground state of the oscillator.

6. Calculate the correctly normalised classical probability to find the particle in the region x and $x + dx$ in the harmonic oscillator potential.

7. Obtain the explicit matrix form for the operators a and $a^\dagger$ in the energy basis.

8. Find $\langle X\rangle, \langle P\rangle, \langle X^2\rangle, \langle P^2\rangle$, and use these to find the uncertainty product $\Delta X \Delta P$ in the n-th state of harmonic oscillator.

9. Show that the representation for the delta function

$$\delta(x - x_0) = \frac{1}{\pi} \lim_{\epsilon \to 0} \frac{\epsilon}{(x - x_0)^2 + \epsilon^2}$$

satisfies the two requirements that $\int \delta(x)dx = 1$ and $\int f(x)\ \delta(x)dx = f(0)$.

10. For the delta function potential given by $V(x) = V_0 \delta(x)$, where $V_0 > 0$, find the eigenfunction and eigenvalues supported by the system. Obtain the reflection and transmission coefficient.

11. Consider the infinite square well potential given by $V(x) = 0$ for $-L < x < L$ and $V(x) = \infty$ for $|x| > L$. A particle in this system is represented by an initial wavefunction at time $t = 0$ by $\phi(x, 0) = A(L^2 - x^2)$, where A is the normalisation constant. Find the probability of obtaining E_n as the energy of the particle, where E_n is the energy of n-th eigenstate of square well system.

12. Using uncertainty principle, find the minimum value of the kinetic energy of a nucleon confined within a nucleus of radius $r = 5 \times 10^{-15}$ m.

14. For the Dirac delta function, prove the following relations;

$$x\,\delta(x) = 0$$

$$\delta(\beta x) = \frac{\delta(x)}{|\beta|}$$

$$\delta(x^2 - a^2) = \frac{1}{2|a|}(\delta(x-a) + \delta(x+a))$$

15. Consider a modified harmonic oscillator for which the potential for $x < 0$ is infinite. For $x > 0$, the potential $V(x) = (1/2)m\omega^2 x^2$. Find the allowed eigenvalues and eigenfunctions.

16. Starting from the generating function for the Hermite polynomial $H_n(x)$, derive the recurrence relation

$$H_{n+1} = 2xH_n(x) - 2nH_{n-1}(x).$$

17. A particle in the infinite square well potential has the initial wave function given by

$$\psi(x, 0) = A(\phi_1(x) + e^{i\theta}\phi_2(x).$$

In this, A is the normalisation constant and θ is the phase factor. At any other time t, find $|\psi(x,t)|^2$ and $\langle x \rangle$.

18. Normalise the state given by,

$$\Psi(x, 0) = A(\phi_0(x) - \phi_1(x))$$

where $\phi_0(x)$ and $\phi_1(x)$ are harmonic oscillator states.

19. Use raising and lowering operators to compute the following matrix elements. Show that

$$\langle 3|X^3|2\rangle = \left(\frac{\hbar}{m\omega}\right)^{3/2} 9\sqrt{3}$$

19. Determine the explicit matrix form of the raising and lowering operators $a^\dagger$ and a.

20. If a represents the lowering operator, project the relation $a|n\rangle = \sqrt{n}|n-1\rangle$ on the X-basis and derive the recursion relation for Hermite polynomials, $H_n^{'}(y) = 2nH_{n-1}(y)$.