PHY313: Quantum Mechanics I : Problem Sheet 1

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(NOTE : *This is a sample selection of problems. You should try out problems from other text books / sources as well. Quantum mechanics textbooks by R. Shankar and Walter Greiner are a good source of problems.*)

1. Starting from $\hat{P}|p\rangle = p|p\rangle$, show that $\langle x|p\rangle = \exp(ixp/\hbar)$.

2. Show that if $\psi(x)$ has mean momentum $\langle p\rangle$, then $e^{ip_0x/\hbar}\psi(x)$ has mean momentum $\langle p\rangle + p_0$.
3. Calculate the matrix elements of $\langle n|X^2|m\rangle$ where $|n\rangle$ and $|m\rangle$ are harmonic oscillator states.

4. Show that in one dimensional problems with the Hamiltonian of the form $\hat{P}^2 + V(x)$, there is no degeneracy in the spectrum.

5. Show that for any normalised $|\psi\rangle$, $\langle\psi|H|\psi\rangle \geq E_0$, where E_0 is the lowest energy eigenvalue.

6. Calculate the correctly normalised classical probability to find the particle in the region x and $x + dx$ in the infinite well potential.

7. Find the energy eigenvalue and the orthogonal eigenstates for $\hat{H} = \hat{p}^2/2m$.

8. Calculate the commutator $[\hat{x}, \hat{p}]$ and the anti-commutator defined as $[\hat{x}, \hat{p}]_+ = \hat{x}\hat{p} + \hat{p}\hat{x}$.

9. Show that the momentum operator in position representation and the position operator in momentum representation are both Hermitian ?

10. Show that for Hermitian operators, the eigenvectors belonging to distinct eigenvalues are orthogonal.

11. If $|x\rangle$ is continuous, show that $\langle x|x'\rangle = \delta(x - x')$.

12. Show that if wavefunction has discontinuities, then it leads to infinite contributions to average kinetic energy of the system.

13. Show that if the derivative of the wavefunction has discontinuities, then it leads to additional contribution to average kinetic energy of the system proportional to the strength of the potential at the point of discontinuity.

14. If $\hat{X}|x\rangle = x|x\rangle$, show that for any potential operator of the form $V(x)$, $V(\hat{X})|x\rangle = V(x)|x\rangle$.

15. Show that $[X, [Y, Z]] + [Y, [Z, X]] + [Z, [X, Y]] = 0$, where X, Y and Z are Hermitian operators. This is called Jacobi identity.

16. What are the dimensions of $\hbar$ and E ?

17. Why do we require low intensity levels in a double slit experiment to infer wave-particle duality ?

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19. If $\psi_n(x)$ represents the n-th eigenstate of the infinite potential well, find the average of momentum operator in these states.

20. Solve the time independent Schrodinger equation for the delta potential $V(x) = V_0\delta(x)$.