PHY313; Test : 3

IISER, Pune. (14 November, 2019)

Time: 50 minutes. Marks : 20.

Answer all the questions. Show all the steps of your calculation clearly.
For sketches, label the axes.

1. If $R_{N,l}(r)$ represents the radial eigenfunction for the hydrogen atom, sketch the following (no calculations needed):
(a) $|R_{1,0}(r)|^2\ r^2$ and
(b) $|R_{2,0}(r)|^2\ r^2$ (4)

2 Derive a formula for the degree of degeneracy of the hydrogen atom in a state with principal quantum number N. What are the reasons for the existence of various degeneracies. (4)

3. For the hydrogen atom in $N = 2, l = 1, m = 0$ state, find the value of $r = r_0$ at which the probability of finding the electron is maximum. (4)

4. Using the L_z operator in (r, θ, ϕ)-basis, solve the eigenvalue problem $L_z|l_z\rangle = l_z|l_z\rangle$. Show and argue out all the steps properly. (4)

5. Starting from $J_\pm|j, m\rangle$, find an explicit matrix form for the operator $J_x - \frac{i}{2}J_y$ in the $j = 1$ subspace alone. Your final answer should be a matrix of the right order for $j = 1$ subspace. (4)

Some useful information :

$$J_\pm|j, m\rangle = \hbar\sqrt{(j \mp m)(j \pm m + 1)}\ |j, m \pm 1\rangle$$