PHY321; Test : 2

IISER, Pune. (29 October, 2019)

Time: 50 minutes. Marks : 20.

Answer all the questions. Show all the steps of your calculation.

For sketches, label the axes.

1. If $|n\rangle$ represents harmonic oscillator states, then the initial state of a particle at time $t = 0$ is given by,

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$

Use raising and lowering operators to find $|\psi(t)\rangle$. (5)

2. If $|\alpha\rangle$ and $|\beta\rangle$ represent two coherent states, show that $|\langle\alpha|\beta\rangle|^2 = \exp\left(-|\alpha - \beta|^2\right)$. (5)

3 Let $U[R(\epsilon, \mathbf{k})]$ be the quantum operator corresponding to infinitesimal rotation by an angle ϵ about z-axis. Starting from $U^\dagger[R]P_yU[R]$, derive the commutator $[P_y, L_z] = i\hbar P_x$. (5)

4. (a) What is the period of the Bloch wave if $kd = (2m+1)(\pi/2)$? (m is integer, d is the periodicity of the potential and k is quasi-momentum).

(b) For Dirac comb with periodicity a, quantisation condition is

$$\cos Ka = \cos \mu a + \frac{m\alpha}{\hbar^2 \mu} \sin \mu a.$$

In this K is the quasi-momentum, α is the strength of the potential and $\mu = \sqrt{2mE/\hbar}$. What are the allowed energy eigenvalues as strength of the potential becomes very large. (1+4)