PHY321; Test : 1

IISER, Pune. (3 Sept, 2019)

Time: 50 minutes. Marks : 25.

Answer all the questions. Show all the steps of your calculation.

For sketches, label the axes.

1. Write the following in Dirac notation (assume ϕ and ψ to be function of x) :

(a) $\int_{-\infty}^{\infty} \widehat{O}_2 \, \phi \, \widehat{O}_1^{\dagger} \, \psi^* \, dx$

(b) $\int_{-\infty}^{\infty} \widehat{O}_1^{\dagger} \phi^* \, \psi \, dx$

(c) $\int_{-\infty}^{\infty} \phi^* \, \phi \, dx$ (5)

2. Starting from ground state, sketch the first three eigenstates $|u(x)|^2$, $(-L \leq x \leq L)$, for the infinite square well potential. (3)

3. Evaluate explicitly the commutator $[\widehat{P_x}, \widehat{X}]$ in position representation. (5)

4. Let an eigenstate be $\psi(x) = A_0 \, e^{ikx} \, e^{-\alpha x}$, and $0 \leq x \leq \infty$, with α and A_0 being positive constants. Calculate $\langle p \rangle$. Explain why $\langle p \rangle$ not a real number. (6)

5. Consider a quantum system whose Hamiltonian is $\widehat{H}$. It has energy eigenvalues E_i and corresponding orthonormal eigenstates $\phi_i(x), i = 1, 2, 3, \ldots N$. An arbitrary state is formed from a superposition given by,

$$\Psi(x, t = 0) = \frac{1}{\sqrt{2}} \, e^{i\alpha_1} \, \phi_1(x) + \frac{a}{\sqrt{3}} \, e^{i\alpha_2} \, \phi_2(x),$$

where α_1, α_2 and a are real and positive numbers.

(i) Find an expression for the state at time T, i.e, find $\Psi(x, t = T)$

(ii) Find the probability that $\Psi(x, t = T)$ is in the state $\phi_2(x)$. (6)