PHY311 : Classical Mechanics

Assignment - 4, (31.10.2016)

Note 1 : This collection of problems have been taken from various sources; text books and other internet sources. You are encouraged to try these problems and many others you might find elsewhere.

Note 2 : This assignment is not meant for evaluation. You should attempt to solve these problems but you need not submit it back.

1. Two identical harmonic oscillators (with masses M and natural frequencies ω_0) are coupled such a way that by adding to the system a mass m common to both oscillators the equations of motion become

$$\begin{aligned} \ddot{x}_1 + (m/M)\ddot{x}_2 + \omega_0^2 x_1 &= 0 \\ \ddot{x}_2 + (m/M)\ddot{x}_1 + \omega_0^2 x_2 &= 0 \end{aligned}$$

2. Consider the case of a double pendulum where the top pendulum has length L_1 and the bottom length is L_2 and similarly the bob masses are m_1 and m_2. The motion is restricted to one plane. Find and describe the normal modes and coordinates. Assume small oscillations.
3. Find the inertia tensor of the cube of density ρ mass M and side of length L in a coordinate system with origin at the centre.

4. Determine the principal axes and principal moments of inertia of a uniformly solid hemisphere of radius r and mass m about its centre of mass.

5. A three-particle system consists of masses m_i and coordinates (x_1, x_2, x_3) as follows ;

$$\begin{aligned} m_1 &= 3m, \qquad (b, 0, b) \\ m_2 &= 4m, \qquad (b, b, -b) \\ m_3 &= 2m, \qquad (-b, b, 0) \end{aligned}$$

Find the inertia tensor, principal axes and principal moments of inertia.

6. If a particle is projected vertically upward to a height h above a point on earth's surface at a northern latitude λ, show that it strikes the ground at a point $\frac{4}{3}\omega \cos\lambda\sqrt{8h^3/g}$ to the west. Neglect air resistance and consider only small vertical heights. In this, ω is the rotational frequency of the earth.

7. Find the coriolis force on an automobile of mass 1300 kg driving north near Fairbanks, Alaska (latitude 65^o) at a speed of 100 km/hour.

8. Show that effect of coriolis force on the motion of Focault pendulum is to rotate the plane of oscillation of the pendulum.