PHY311 : Classical Mechanics

Assignment - 3, (7.10.2016)

Note 1 : This collection of problems have been taken from various sources; text books and other internet sources. You are encouraged to try these problems and many others you might find elsewhere.

Note 2 : This assignment is not meant for evaluation. You should attempt to solve these problems but you need not submit it back.

1. Show by explicit calculation that the problem of two bodies with masses m_1 and m_2 moving in a central force field can be reduced to an equivalent single particle problem.

2. Can a repelling force field, $f(r) > 0$, lead to circular orbits ? Explain your reason.

3. For a particle moving under a central force of the type $F(r) = -Kr^4$ find the radius and energy of circular orbits.

4. Innvestigate the motion of a particle repelled by a force centre according to the force law $F(r) = kr$, where k is a constant. For this case, show that the orbit is hyperbolic.

5. A particle moves in an elliptical orbit in an inverse-square-law central force field. If the ratio of the maximum angular velocity to the minimum angular velocity of the particle is n, show that the eccentricity of the orbit is

$$\epsilon = \frac{\sqrt{n}-1}{\sqrt{n}+1}.$$

6. Assume earth's orbit to be circular. The mass of the sun suddenly decreases to half its original value. What is new orbit of the earth ? Will the earth escape out of solar system ?

7. A particle of unit mass moves from infinity along a straight line that, if continued, would allow it to pass a distance $b\sqrt{2}$ from a point P. If the particle is attracted toward P with a force varying as k/r^5, and if the angular momentum about the point P is $\sqrt{k}/b$, show that the trajectory is given by,

$$r = b \ \coth\left(\frac{\theta}{\sqrt{2}}\right)$$

8. Find the force law for a central-force field that allows a particle to move in a logarithmic spiral orbit given by $r = ke^{\alpha\theta}$, where k and α are constants.

9. A particle moves in a central force field given by the potential

$$V = -k\frac{e^{-ar}}{r},$$

where k and a are positive constants. What is the equivalent one dimensional problem. Determine the nature of motion (whether it is circular, elliptic, hyperbolic etc.) and the range of angular momentum and energy for which each of them is valid.

10. Calculate the differential cross-section $\sigma(\theta)$ for the elastic scattering of a particle from an impenetrable sphere; the potential is given by,

$$\begin{aligned} U(r) &= 0, \qquad r > a, \\ &= \infty, \qquad r < a, \end{aligned}$$

11. A fixed force centre scatters a particle of mass m as per the force law $F(r) = k/r^3$. If the initial velocity of the particle is u_0, show that the differential cross section is

$$\sigma(\Theta) = \frac{k\pi^2(\pi - \Theta)}{mu_0^2\Theta^2(2\pi - \Theta)^2 \sin\Theta}$$

12. For Rutherford scattering, show that the distance of closest approach of an incoming particle to the scattering center is given by

$$d = \frac{b}{2}\left(1 + \frac{1}{\sin\frac{\Theta}{2}}\right)$$

where b is the distance of closest approach for zero impact parameter, i.e., (head-on collision).